Finite Width and Asymptotic Regge Trajectories of Quark Gluon Bags

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Outline

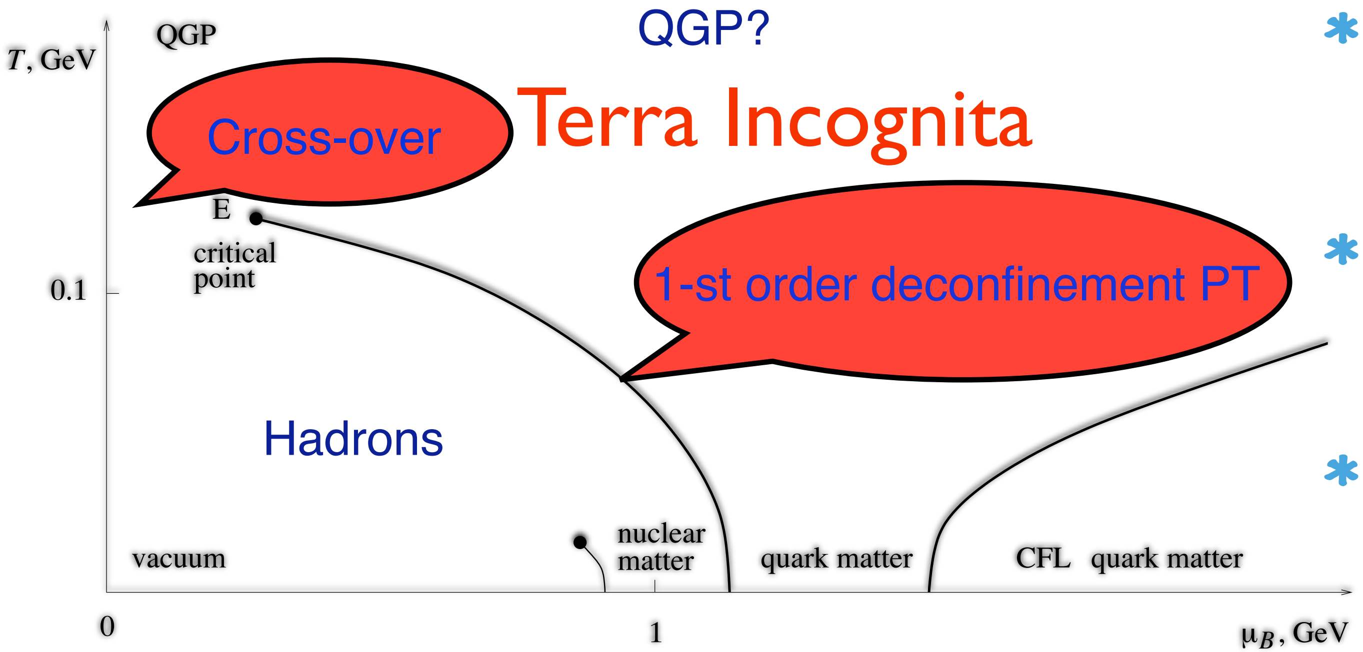
- Two Conceptual Problems of statistical models of QGP Bags
- The Finite Width Model (FWM) of the QGP bags with Hagedorn spectrum
- Derivation of the QGP Bag pressure at T > T_H
- QGP pressure at T ~ 0 and the Subthreshold Suppression of QGP Bags
- Lattice QCD data and the width estimate for the QGP Bags
- Regge trajectories of free QGP bags, and QGP bags at high T and low T
- Conclusions

K.A.B., V.K.Petrov, G.M.Zinovjev, Europhys. Lett. 85 (2009);
arXiv:0801.4869 and PRC 79 (2009);

K.A.B., PRC 76 (2007); arXiv:0809.1023

25 Years of QGP Searches in HIC

- AGS (BNL), SPS (CERN) & RHIC (BNL) - many nice findings, BUT no smoking gun for QGP!!!
- SPS (CERN), RHIC low energy program (BNL) - searches for the (tri)critical endpoint of QCD phase diagram;
- NICA (JINR, Dubna) - searches for the mixed phase (hadrons+QGP);
- FAIR (GSI, Darmstadt) - searches for the densest state of nuclear matter



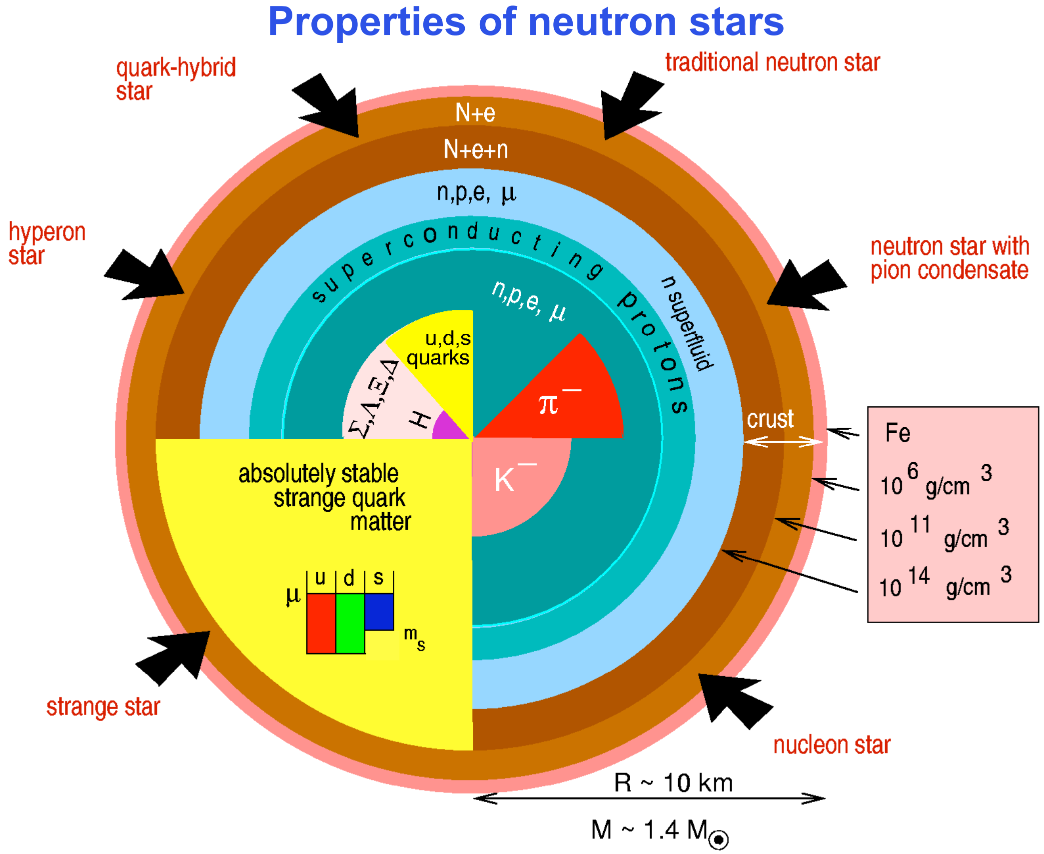
Phase diagram major elements:

- 1-st order deconfinement PT (low T, large μ)
- cross-over transition (low μ, large T)
- (tri)critical endpoint in between. **Exact location is unknown!**

Astro & Cosmic QGP Searches Programs

- Quark (core) stars, neutron stars, **stable strange stars, ...**

- **Strangelet** - {see Bodmer (1971), Witten(1984), Jaffe (1984)} finite drop of strange matter with **LARGE** baryonic charge and **small** electric charge. May be stable at high densities (few normal nuclear densities) due to Chiral Symmetry (CS) Restoration {Buballa(1996}: In CS **Restored** phase s-quark mass << Fermi energy of u & d quarks => u & d quarks weakly decay into s-quarks!



They can be formed is A+A HIC, in QCD phase transition in early Universe, in collisions of compact stars with large strangeness, in cosmic rays e.t.c.

First Conceptual Problem

- Why the small and not too heavy QGP bags with mass of 10–20 GeV have not been observed in A+A or in elementary particle collisions at low T?
- Why the strangelets were never observed at low T?

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- Why the small and not too heavy QGP bags with mass of 10–20 GeV have not been observed in A+A or in elementary particle collisions at low T?
- Why the strangelets were never observed at low T?

* Usual concept: QGP bags cannot exist inside hadronic phase because of PT or strong cross-over. They should be extremely suppressed statistically.

Usual Concept: Gas of Bags Model

- Why the small and not too heavy QGP bags with mass of 10–20 GeV have not been observed in A+A or in elementary particle collisions?

OLD CONCEPT (for $\mu = 0$):
at low $T < T_c$ the QGP bags **cannot coexist** with hadrons due to phase transition (or strong cross-over).

$$\underbrace{\hat{Z}(s,T)}_{\text{Isobaric partition}} \equiv \int_0^\infty dV \exp(-sV) \underbrace{Z(V,T)}_{\text{GCE partition}} = \frac{1}{[s - F(s,T)]},$$

Simple pole $\underbrace{s^* = s_H(T)}_{\text{HADRONIC}}$ **and essential singularity** $\underbrace{s^* = s_Q(T)}_{\text{QGP}}$: $\quad s^* = F(s^*, T)$

$$\phi(T, m_k) \equiv \frac{1}{2\pi^2} \int_0^\infty p^2 dp\, e^{-\frac{(p^2 + m_k^2)^{\frac{1}{2}}}{T}}$$

DEFINE PHASES

Typical form of bag spectrum: the **discrete mass-volume spectrum** $F_H(s,T)$ of hadrons lighter than M_0 and the **continuous volume spectrum** $F_Q(s,T)$

$$F(s,T) \equiv F_H(s,T) + F_Q(s,T) =$$

$$\sum^n g_j e^{-v_j s} \phi(T, m_j) + \int_{V_0}^\infty dv \int_{M_0}^\infty dm \int \frac{d^3k}{(2\pi)^3} \rho(m,v)\, e^{-sv - \frac{\sqrt{k^2+m^2}}{T}} =$$

Hard core repulsion

Hard core repulsion

$$\sum_{j=1} g_j e^{-v_j s} \phi(T, m_j) + u(T) \int_{V_0}^\infty dv \frac{\exp[(s_Q(T) - s)\, v]}{v^\tau},$$

$V_0 \approx 1\ \mathrm{fm}^3$

$s_Q(T) = \frac{p_Q(T)}{T}$ is defined via the QGP pressure $p_Q(T)$ (MIT Bag Model). $\quad M_0 \approx 2.5$ GeV

$\Rightarrow$ In hadronic phase $s = s_H(T) > s_Q(T)$

$\Rightarrow$ LARGE QGP BAGS in the spectrum $F(s,T)$ are **exponentially suppressed!**

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- Why the small and not too heavy QGP bags with mass of 10–20 GeV have not been observed in A+A or in elementary particle collisions?

OLD CONCEPT (for $\mu = 0$):
at low $T < T_c$ the QGP bags **cannot coexist** with hadrons due to phase transition (or strong cross-over).

> * However, this is true for an infinite system only! In finite systems the suppression is not of Avogadro number order, but 1/10000 – 1/100000 only! Then such QGP bags (and strangelets!) should have been observed as any METASTABLE STATE!
>
> * If they are absent, then there must be a reason for this!

$$\sum^{n} g_j e^{-v_j s}\phi(T,m_j) \quad + \int_{V_0}^{\infty} dv \int_{M_0}^{\infty} dm \int \frac{d^3k}{(2\pi)^3}\, \rho(m,v)\, e^{-sv-\frac{\sqrt{k^2+m^2}}{T}} \quad =$$

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$$\sum_{j=1} g_j e^{-v_j s}\phi(T,m_j) \quad + \quad u(T) \int_{V_0}^{\infty} dv \, \frac{\exp\left[(s_Q(T)-s)\, v\right]}{v^{\tau}} \,, \qquad \mathbf{V_0 \approx 1\ fm^3}$$

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Hagedorn Mass Spectrum

GBM contains the Hagedorn mass (volume) spectrum of bags

PARADOXICAL SITUATION WITH THE HAGEDORN MASS SPECTRUM:

$$\rho(m)\Big|_{m \gg T_H} \sim \exp\left[\frac{m}{T_H}\right]$$

It was predicted for $m \gg 1$ GeV by Hagedorn in 1965

It follows from the statistical bootstrap model (Frautschi, 1971);

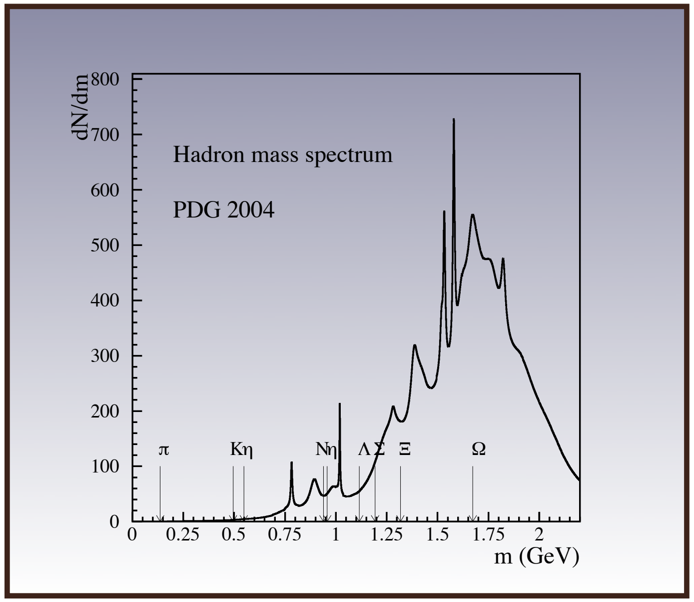
from Veneziano model (1970), from Bag Model (Kapusta, 1981);

from large N_c limit of 3+1 QCD (Cohen, 2009)

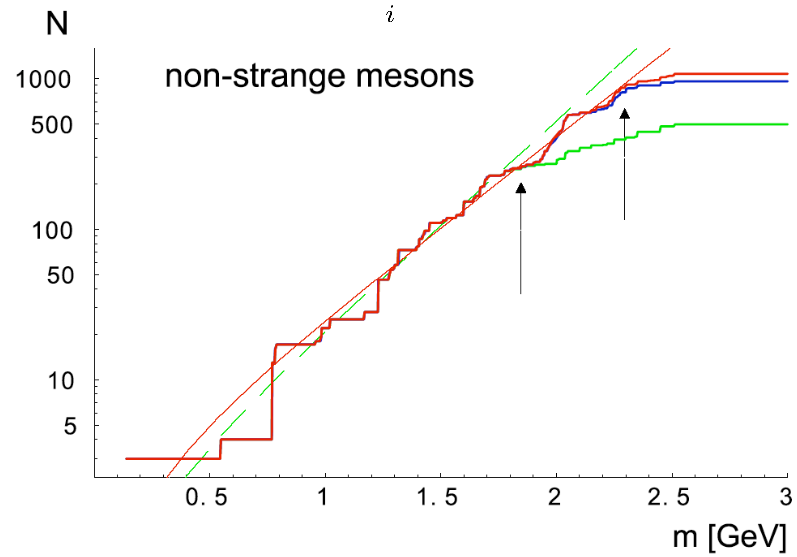
Also the Hagedorn mass spectrum is observed experimentally,

BUT

Second Conceptual Problem



$$N_{\mathrm{exp}}(m) = \sum_i g_i \Theta(m - m_i),$$



It is observed for 1.3 GeV $< m <$ 2.5 GeV only,

i.e. NOT WHERE IT WAS PREDICTED!

$\Rightarrow$ **There is a huge deficit of heavy hadrons predicted by stat. bootstrap model!**

It is believed that heavy resonances are not observed due to their large width.

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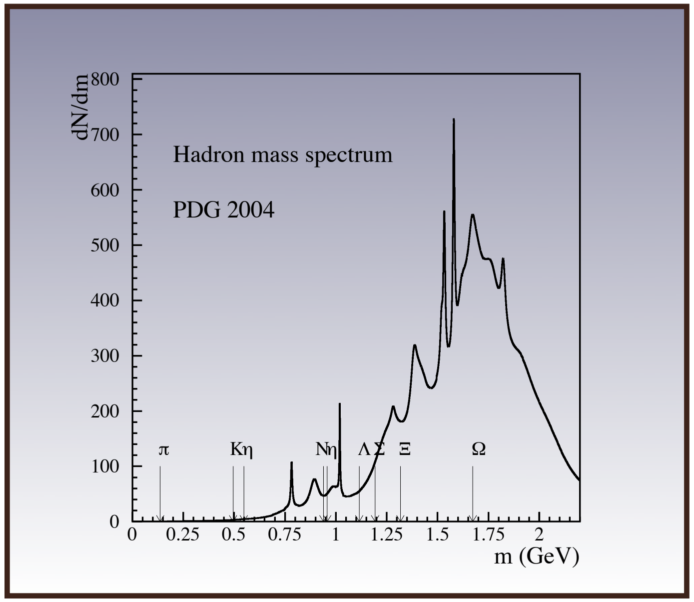

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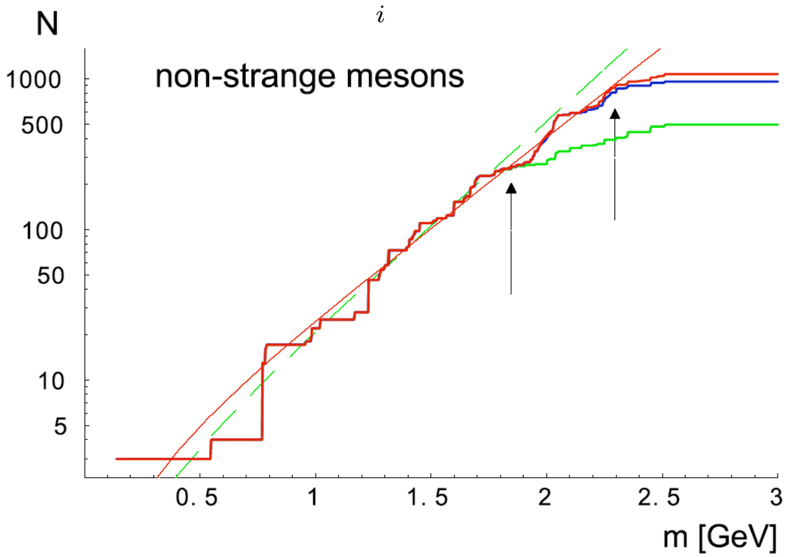

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It is believed that heavy resonances are not observed due to their large width.

However, the full Hagedorn mass spectrum is used in ALL realistic statistical models like Gas of Bags Model (GBM) and NO width is accounted for!

For width of QGP bags see D.Blaschke & K.A.B. in 2003-2005

Finite Width Model

Major aims are:

1) to include the finite medium dependent width into statistical model in the most general fashion (FWM).

2) to resolve these two conceptual problems and to derive a general form of EOS from the clear physical assumptions.

3) to compare the obtained EOS with the lattice QCD results and to find out the width of heavy QGP bags.

In fact, we want to make a firm bridge between the lattice QCD thermodynamics and hadronic phenomenology via the statistical approach.

Finite Width Model Spectrum

- To make the bridge from hadronic phenomenology **we need the Hagedorn-like mass spectrum!**

Experimental input

- To introduce the **width Γ we need the Gaussian attenuation,** since **the Breit-Wigner one does not work** for the Hagedorn-like mass spectrum!

To have convergent partition

- To get a realistic model **we need to introduce the surface tension for QGP bags!**

To have (tri)critical endpoint

$\Rightarrow$ **the simplest parameterization of the spectrum** $\rho(m,v)$ **is**

$$\rho(m,v) = \frac{\rho_1(v)\ N_\Gamma}{\Gamma(v)\ m^{a+\frac{3}{2}}} \exp\ \underbrace{\left[\frac{m}{T_H} - \frac{(m-Bv)^2}{2\Gamma^2(v)}\right]}_{\text{Hagedorn \& Gaussian terms}}, \quad \text{with} \quad \rho_1(v) = f(T)\, v^{-b}\ \exp \underbrace{\left[-\frac{\sigma(T)}{T}\, v^{\varkappa}\right]}_{\text{Surface tension}},$$

K.A.B. PRC76(2007)

Important:

- **Gaussian width $\Gamma(v)$ depends on bag's volume v, on T, but not on mass m!**
- Gaussian width $\Gamma(v)$ is related to the true resonance width as $\Gamma_R = 2\sqrt{2\ln 2}\,\Gamma(v) \approx 2.355\,\Gamma(v)$
- The **most probable mass in a vacuum** must be positive $B > 0$

$\Rightarrow$ Normalization factor is

$$N_\Gamma^{-1} = \int\limits_{M_0}^{\infty} \frac{dm}{\Gamma(v)} \exp\left[-\frac{(m-Bv)^2}{2\Gamma^2(v)}\right] \approx \Big|_{v \gg V_0, M_0/B} \approx \sqrt{2\,\pi}$$

Analysis of the FWM Spectrum

• For simplicity let's consider **only two choices** for Gaussian width $\Gamma(v)$:

v-**independent width** $\Gamma(v) = \mathbf{\Gamma_0} \equiv Const$ and v-**dependent width** $\Gamma(v) = \mathbf{\Gamma_1} \equiv \gamma \mathbf{v}^{\frac{1}{2}}$

Ignoring the hard-core repulsion and thermostate in $F_Q(s,T)$**:** $\Rightarrow$

$$\rho(m) \equiv \int_{V_0}^{\infty} dv\, \rho(m,v) = \int_{V_0}^{\infty} dv\, \frac{\rho_1(v)\ N_\Gamma}{\Gamma(v)\ m^{a+\frac{3}{2}}} \exp\left[\frac{m}{T_H} - \frac{(m-Bv)^2}{2\Gamma^2(v)}\right] \approx \Big|_{m \gg M_0} \approx \frac{\rho_1(\frac{m}{B})}{B\ m^{a+\frac{3}{2}}} \exp\left[\frac{m}{T_H}\right]$$

Can be derived, if for $v \gg V_0$ the **width grows slower than** $\mathbf{v}^{(1-\varkappa/2)} = \mathbf{v}^{2/3}$

This is so, since for $\Gamma(v) = \mathbf{\Gamma_0}$ or $\Gamma(v) = \mathbf{\Gamma_1}$ the Gaussian width acts like the Dirac δ-function!

$\Rightarrow$ **The FWM spectrum corresponds to the Hagedorn mass spectrum** modified by the surface tension!

$\Rightarrow$ Similarly, the mean width $\overline{\Gamma(v)} \approx \Gamma(m/B)$

$\Rightarrow$ for $\Gamma(v) = \mathbf{\Gamma_1}$ one gets **the large mean width** $\Gamma_1(m/B) = \gamma\sqrt{m/B}$

$\Rightarrow$ **for** $\mathbf{\Gamma_1(m/B)} = \gamma\sqrt{\mathbf{m/B}}$ **the heavy resonances are hard to be observed!**

THE SECOND CONCEPTUAL PROBLEM IS RESOLVED.

High T Behavior of FWM Spectrum

Let's calculate $F(s,T)$. Depending on the sign of the **most probable mass**

$$\underbrace{\langle m \rangle \equiv Bv + \Gamma^2(v)\beta}_{\textbf{most probable mass}}, \quad \text{with} \quad \beta \equiv T_H^{-1} - T^{-1}$$

there are two distinct cases : $\langle m \rangle > 0$ and $\langle m \rangle < 0$

Let's calculate $F(s,T)$ for $T \geq T_H$ $\Rightarrow$ $\boldsymbol{\langle m \rangle > 0}$ for $v \gg V_0$ **by saddle point**

For $M_0 \gg T$ one can use the nonrelativistic approximation for momentum $\Rightarrow$

$$F_Q^+(s,T) = \int_{V_0}^{\infty} dv \int_{M_0}^{\infty} dm \int \frac{d^3k}{(2\pi)^3}\, \rho(m,v)\, e^{-\mathbf{s}v - \frac{\sqrt{k^2+m^2}}{T}} = \left[\frac{T}{2\pi}\right]^{\frac{3}{2}} \int_{V_0}^{\infty} dv \int_{M_0}^{\infty} dm\, \frac{\rho_1(v)\; N_\Gamma}{\Gamma(v)\; m^a} \exp\left[\beta m - \frac{(m-Bv)^2}{2\Gamma^2(v)} - sv\right]$$

$$F_Q^+(s,T) \approx \left[\frac{T}{2\pi}\right]^{\frac{3}{2}} \int_{V_0}^{\infty} dv\, \frac{\rho_1(v)}{\langle m \rangle^a} \exp\left[\frac{(p^+ - sT)v}{T}\right],$$ **with the pressure**

$$p^+ \equiv T\left(\beta B + \frac{\Gamma^2(v)}{2v}\beta^2\right)$$

In terms of $\boldsymbol{\langle m \rangle > 0}$ it reads as: $\boldsymbol{p^+ \equiv \frac{T\beta}{v}\left[\langle m \rangle - \frac{1}{2}\Gamma^2(v)\beta\right]}$

$\Rightarrow$ **In general, the pressure of large QGP bags is due to the mass density and the width!**

Note that width may CORRECTLY contribute into $\boldsymbol{\langle m \rangle > 0}$ and $\boldsymbol{p^+}$ for $\boldsymbol{\Gamma(v) \leq \Gamma_1}$ only!

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there are two distinct cases : $\langle m \rangle > 0$ and $\langle m \rangle < 0$

Let's calculate $F(s,T)$ for $T \geq T_H$ $\Rightarrow$ $\langle \boldsymbol{m} \rangle > \mathbf{0}$ for $v \gg V_0$ **by saddle point**

* However, this case does not resolve the first problem!

$$F_Q^+(s,T) = \int_{V_0}^{\infty} dv \int_{M_0}^{\infty} dm \int \frac{d^3k}{(2\pi)^3}\, \rho(m,v)\, e^{-\mathbf{s}v - \frac{\sqrt{k^2+m^2}}{T}} = \left[\frac{T}{2\pi}\right]^{\frac{3}{2}} \int_{V_0}^{\infty} dv \int_{M_0}^{\infty} dm\, \frac{\rho_1(v)\ N_\Gamma}{\Gamma(v)\ m^a} \exp\left[\beta m - \frac{(m-Bv)^2}{2\Gamma^2(v)} - sv\right]$$

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with the pressure

$$p^+ \equiv T\left(\beta B + \frac{\Gamma^2(v)}{2v}\beta^2\right)$$

In terms of $\langle \boldsymbol{m} \rangle > \mathbf{0}$ it reads as: $\boldsymbol{p^+ \equiv \frac{T\beta}{v}\left[\langle m \rangle - \frac{1}{2}\Gamma^2(v)\beta\right]}$

$\Rightarrow$ **In general, the pressure of large QGP bags is due to the mass density and the width!**

Note that width may CORRECTLY contribute into $\langle \boldsymbol{m} \rangle > \mathbf{0}$ and $\boldsymbol{p^+}$ for $\boldsymbol{\Gamma(v) \leq \Gamma_1}$ only!

Low T Behavior of FWM Spectrum

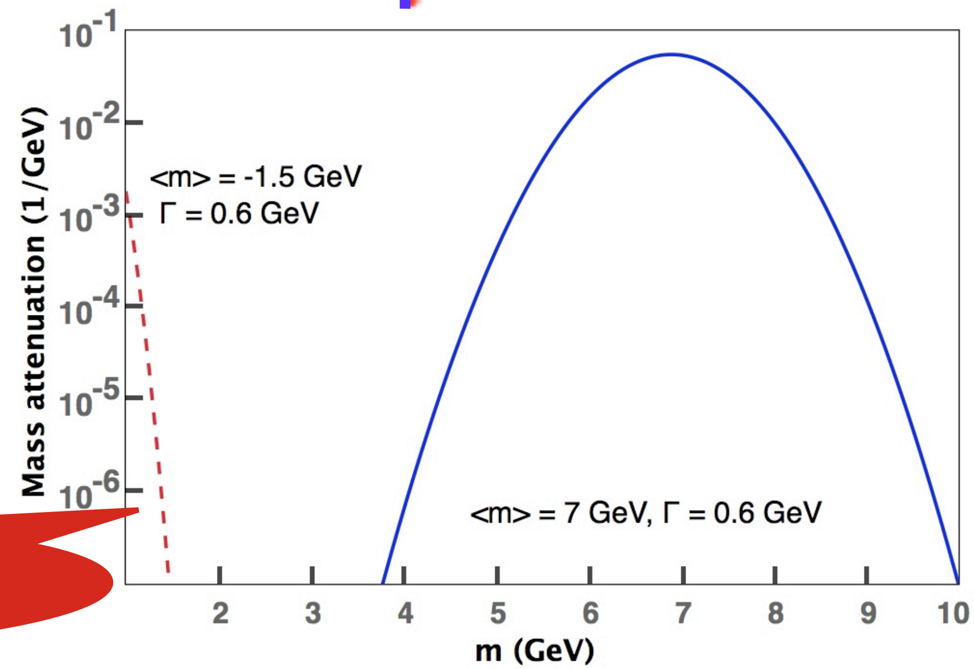
For $T \ll T_H$ and $0 < B < \infty \quad \Rightarrow$

$\langle \boldsymbol{m} \rangle < \mathbf{0}$ for $v \gg V_0$: **by steepest descent!**

The maximum is below M_0 and ,hence,

the tail of distribution contributes only!

$\Rightarrow$ **Subthreshold Suppression of QGP bags**



$$F_Q^-(s,T) = \int_{V_0}^{\infty} dv \int_{M_0}^{\infty} dm \int \frac{d^3k}{(2\pi)^3}\, \rho(m,v)\, e^{-sv - \frac{\sqrt{k^2+m^2}}{T}} = \left[\frac{T}{2\pi}\right]^{\frac{3}{2}} \int_{V_0}^{\infty} dv \int_{M_0}^{\infty} dm\, \frac{\rho_1(v)\ N_\Gamma}{\Gamma(v)\ m^a} \exp\left[\beta m - \frac{(m - Bv)^2}{2\Gamma^2(v)} - sv\right]$$

$$F_Q^-(s,T) \approx \left[\frac{T}{2\pi}\right]^{\frac{3}{2}} \int_{V_0}^{\infty} dv \frac{\rho_1(v) N_\Gamma\, \Gamma(v)\, \exp\left[\frac{(p^- - sT)v}{T}\right]}{M_0^a\, [M_0 - \langle m \rangle + a\, \Gamma^2(v)/M_0]}\,,$$

with the pressure

$$p^- = \frac{T}{v}\left[\beta M_0 - \frac{(M_0 - Bv)^2}{2\,\Gamma^2(v)}\right] \approx \Big|_{v \gg V_0} \approx -T\,\frac{B^2}{2\,\gamma^2}\,.$$

Important: • Can be derived for $\boldsymbol{B} > \mathbf{0}$ only!

if $\boldsymbol{B} < \mathbf{0}$, then $N_\Gamma \approx [M_0 - \langle m \rangle]\, \Gamma^{-1}(v) \exp\left[\frac{(M_0 - Bv)^2}{2\,\Gamma^2(v)}\right]$ would cancel the leading term in $\boldsymbol{p}^-$

• Can be derived for $\boldsymbol{\Gamma(v) = \Gamma_1(v)}$, since only in this case $\langle m \rangle \equiv Bv + \Gamma^2(v)\beta < 0$ for $B > 0$ at low T!

Volume Dependence of Width

- **The case** $\langle m \rangle \equiv Bv + \Gamma^2(v)\beta > 0$ exists for $T \geq \alpha T_H$ with $\alpha < 1$

It generates the QGP bag pressure $p^+ \equiv T\left(\beta B + \frac{\Gamma^2(v)}{2v}\beta^2\right) = \frac{T\beta}{v}\left[\langle m \rangle - \frac{1}{2}\Gamma^2(v)\beta\right]$

for any $\Gamma(v)$, if **width grows slower than** $\mathbf{v}^{(1-\varkappa/2)} = \mathbf{v}^{2/3}$, but is **meaningful for** $\Gamma(\mathbf{v}) \leq \Gamma_1(\mathbf{v})$

HAS THE SAME PHASE STRUCTURE AS THE QGBSTM WITH $\tau = a + b$.

K.A.B. PRC76(2007)

- **The case** $\langle m \rangle \equiv Bv + \Gamma^2(v)\beta < 0$ exists for $T < \alpha T_H$ with $\alpha < 1$

It generates the QGP bag pressure $p^- = -T\frac{B^2}{2\gamma^2}$ and **can be derived for** $\Gamma(v) = \Gamma_1(v)$ **only!**

$\Rightarrow$ **This is truly nonperturbative effect because for stable hadrons it does not exist!**

Closely resembles low T pressure known from lattice QCD!!!

$\Rightarrow$ **Finite pressure of large QGP bags with nonzero width exists for** $\Gamma(v) = \Gamma_1(v) = \gamma\sqrt{v}$ **only!**

First Conceptual Problem is Resolved

For $T < \alpha T_H$ with $\alpha < 1$ exists Subthreshold Suppression of QGP bags $\Rightarrow$

only the bags with mass $\sim M_0 \approx 2.5$ GeV and volume $\sim V_0 \approx 1$ fm^3

could contribute into partition, but they are HIGHLY SUPPRESSED!

$\Rightarrow$ Case $\langle m \rangle < 0$ resolves the first conceptual problem for finite systems for $T < \alpha T_H$.

What about $\alpha T_H \leq T \leq T_H$?

Now I show that in this case the QGP bags have very large width,

$\Rightarrow$ they are indistinguishable from the usual short-living hadrons!

MIT Bag Model EoS

- **Consider stable resonances $\Gamma(v) = 0$ and MIT bag model**

Equating $\boldsymbol{p^+ = T\beta B}$ with $\boldsymbol{p_{bag} \equiv \sigma T^4 - B_{bag}} \quad \Rightarrow \quad \boldsymbol{B_{bag} = \sigma T_H^4}$ and

the most probable mass density $\quad \boldsymbol{\frac{\langle m \rangle}{v} \equiv B = \sigma T_H (T + T_H)(T^2 + T_H^2)}$

$\Rightarrow \quad \boldsymbol{p^+ = T\beta B}$ **and** $\boldsymbol{\langle m \rangle = Bv > 0} \Rightarrow$ **there is no subthreshold suppression.**

$\Rightarrow$ **The usual MIT bag model can be reproduced.**

However, it has both conceptual problems!

Keep in mind: $\quad \boldsymbol{B_{MIT} = \sigma T_H (T_H^3 + T_H^2 T + T_H T^2 + T^3)}$

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However, it has both conceptual problems!

Keep in mind: $B_{MIT} = \sigma T_H (T_H^3 + T_H^2 T + T_H T^2 + T^3)$

Moreover, MIT Bag Model EoS contradicts to LQCD!

More Realistic EoS

• The model pressure $p_a = \sigma T^4 - A_1 T \quad (A_1 > 0)$

describes LQCD data well: C. G. Kallman, Phys. Lett. B 134, 363 (1984),

M. I. Gorenstein, O. A. Mogilevsky, Z. Phys. C 38 (1988)

But these are OLD LQCD data!

For new data analysis see K.A.B. et al. PRC 79 (2009)

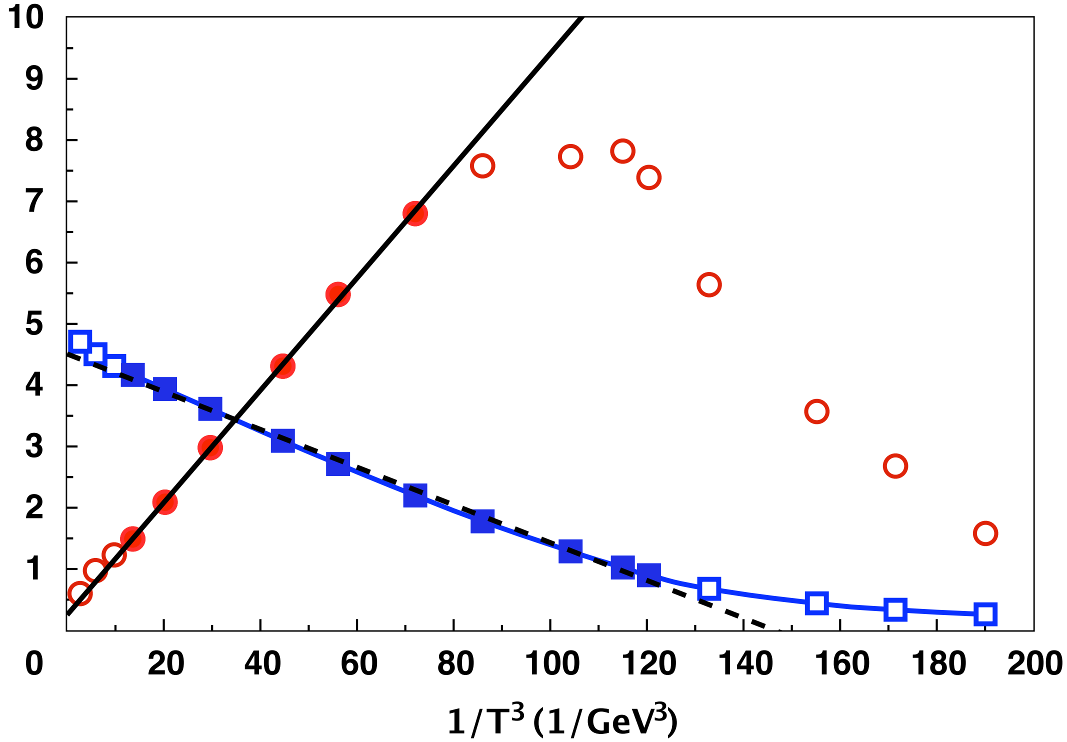
Entropy density: $s = \frac{dp}{dT} = 4\sigma T^3 - A_1;$

and energy density: $\varepsilon = Ts - p = 3\sigma T^4$ $\Rightarrow$ NO T-linear term in ε!

$$\Rightarrow \frac{p}{T^4} = \sigma - \frac{A_1}{T^3}, \qquad \frac{\delta}{T^4} = \frac{\varepsilon - 3\,p}{T^4} = \frac{3A_1}{T^3}$$

$\Rightarrow$ are linear in $\frac{1}{T^3}$

Width Estimate from Lattice QCD



recent LQCD data:

$SU(3)_C$ with 3 flavors

Cheng et al, arXiv:0710.0354

Red symbols:

Trace anomaly $\delta/T^4 = (\varepsilon - 3\,p)/T^4$

χ^2/d.o.f ≈ 0.062

OUR fit: filled symbols

Blue symbols: p/T^4

$\Rightarrow$ LQCD pressure has σT^4, $-\sigma T_H^3 T$ and $T^4 \ln \frac{T}{T_H}$ terms!

Obtained by LQCD data fit

$$p_{QGP} = \sigma\, T^4 \boxed{-\,\sigma T_H^3\, T} + \underbrace{\tilde{a}_0 T^4 \ln\left[\frac{T}{T_H}\right]}_{small}, \qquad \text{for} \qquad 240\,\text{MeV} \leq T \leq 420\,\text{MeV}$$

COMPARE T-linear terms!

$$p^- = \boxed{-T\,\frac{B^2}{2\,\gamma^2}}$$

Derived by FWM at low T!

Comparison with Lattice QCD

- Consider the finite width of resonances $\Gamma(v) = \Gamma_1(v)$ and the model pressure

$p_a = \sigma T^4 - A_1 T$: for $T \to \infty \Rightarrow p_a \to \sigma T^4$,
but for $T \to 0 \Rightarrow p_a \to -A_1 T$

Compare this with
$p^+ = T\left[\beta B + \frac{1}{2}\gamma^2\beta^2\right]$ for $T \geq \alpha T_H$ and $p^- = -T\frac{B^2}{2\gamma^2}$ for $T < \alpha T_H$

$\Rightarrow$ Idea: for all $T \leq \alpha T_H \Rightarrow B(T) \approx B_{MIT}$ (up to small correction T^3)

$\Longleftrightarrow$ For vanishing width the QGP bag pressure is MIT Bag model pressure,

while finite width generates T-linear pressure!

$\Rightarrow B(T) = \sigma T_H^2 (T_H^2 + T_H T + T^2)$ for any $T \leq T_H$

$$\Rightarrow \quad \gamma^2 = 2T\sigma T_H^2 (T^2 + TT_H + T_H^2) \equiv 2T\, B(T)$$

$$\Rightarrow \quad \frac{\langle m \rangle}{v} = B(T)\left[\frac{2T}{T_H} - 1\right] \; ; \quad \alpha = 0.5$$

$$\Rightarrow \quad \gamma_0^2 = \frac{B_0^2}{2A_1} = \frac{1}{2}T_H B_0 = \frac{\sigma}{2}T_H^5$$

$$\Rightarrow \quad \min \Gamma_1(v) = \Gamma_1(V_0, T=0) \approx \left[\frac{\sigma}{2}V_0\right]^{\frac{1}{2}} T_H^{\frac{5}{2}}$$

Width Estimate Sensitivity

TABLE I: The values of the resonance width for different models. Model A corresponds to the $SU(2)_C$ pure gluodynamics of Ref. [45]. Model B describes the $SU(3)_C$ LQCD with 2 quark flavors [46] and Model C is the $SU(3)_C$ LQCD with 3 quark flavors [50].

$$V_0 = 1\ \mathrm{fm}^3$$

	Model Ref.	$\frac{90\sigma}{\pi^2}$ d.o.f.	T_c (MeV)	$\Gamma_R(V_0, 0)$ (MeV) at T=0	$\Gamma_R(V_0, T_H)$ (MeV) at T=Th	
SU(2)c pure gluodynamics	A	6	170	410	1420	Bielefeld data, finite-size effects are accounted for
	A	6	200	616	2133	
SU(3)c LQCD 2 q flavors	B	37	170	391	1355	Bielefeld data, finite-size effects are accounted for
	B	37	200	587	2034	
SU(3)c LQCD 3 q flavors	C	$\frac{95}{2}$	196	596	2066	Bielefeld+BNL+ Copenhagen data, no FSE, but large lattices!

Width Estimate Sensitivity

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SU(2)c pure gluodynamics — Bielefeld data, finite-size effects are accounted for

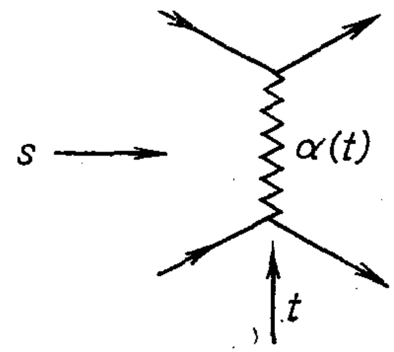
SU(3)c LQCD 2 q flavors — Bielefeld data, finite-size effects are accounted for

SU(3)c LQCD 3 q flavors — Bielefeld+BNL+ Copenhagen data, ... ices!

- Width of QGP bags is very stable against dof number! Strong argument in favor of B-ansatz.
- Strongly depends on T and at Tc!
- QGP bags with so large width cannot be observed!

What is the Regge trajectory

For such h + h into h + h reaction
the amplitude A is



$$A(S,t) \;=\; S^{\alpha(t)}$$

I.e. there is an exchange of the trajectory!

Here S is invar. mass square, and t is momentum transfered square

Meaning: in complex S-plane (physical one) $\Rightarrow$

$\alpha(S_r) = J$ is a resonance spin at its position $S = S_r$!

Can We relate FWM to Regge trajectories?

Asymptotic behavior $|\boldsymbol{S_r}| \to \infty$ of the Regge trajectories was studied in detail by Trushevsky, UJP 22 (1977)

S_r is Invariant Mass Square of 2 hadrons $\Rightarrow$ $S_r > 4$ Mass^2

It was found an asymptotic behavior $\boldsymbol{\alpha(S_r) = -\gamma_r[-S_r]^\nu \cdot f(S_r)}$ with $\frac{1}{2} \leq \nu \leq 1$ only!

Here $\gamma_r = const > 0$, **for** $|S_r| \to \infty$ **function** $f(S_r)$ **must grow slower than any power of** $|S_r|$

Writing $S_r = |S_r| e^{i\phi}$ and energy $\boldsymbol{\sqrt{S_r} = M_r - \frac{i}{2}\Gamma_r}$

$\Rightarrow$ $\boldsymbol{\frac{\Gamma_r}{M_r} = -2\, tg\left(\frac{\phi}{2}\right)}$ at the line $\mathbf{Im}\ \boldsymbol{\alpha(S_r) = 0}$

$\alpha(S_r)$ RELATES WIDTH AND MASS, $\Rightarrow$ IT WOULD BE NICE TO COMPARE IT WITH FWM RESULTS!

Directly this is impossible since in FWM v (i.e. $\Gamma(v)$) and m of bag are independent variables!

Indirectly, this **can be done for the mean mass.**

$$\overline{m}(v) \equiv \int\limits_{M_0}^{\infty} dm \int \frac{d^3k}{(2\pi)^3}\, \rho(m,v)\ m\ e^{-\frac{\sqrt{k^2+m^2}}{T}} \left[\int\limits_{M_0}^{\infty} dm \int \frac{d^3k}{(2\pi)^3}\, \rho(m,v)\ e^{-\frac{\sqrt{k^2+m^2}}{T}} \right]^{-1}$$

Note, another way of averaging is technically bit more complicated!

Regge trajectory of free QGP bags

Use averaging with respect to volume for free bags (**no thermostate, no interaction**):

$$\Rightarrow \quad \overline{\Gamma_1(v)} \approx \Gamma_1(m/B) = \gamma \sqrt{\frac{m}{B}}$$

$$\text{For} \quad \Gamma_1(m) \to \infty \quad \Rightarrow \quad \frac{\Gamma_1(m)}{m} = \frac{\gamma}{\sqrt{B\,m}} \to 0$$

This exactly corresponds to the upper limit of the Regge trajectory behavior $\nu = 1$!

Moreover, Trushevsky's example $\alpha(S_r) = \gamma_r[S_r + a(-S_r)^p]$ with $a < 0$ and $p = \frac{3}{4}$

exactly corresponds to FWM free bags with $\Gamma_r = 2\sqrt{2\ln 2}\,\Gamma_1(m)$ and $|S_r| \approx m^2$:

$$\text{From } \alpha(S_r)\Big|_{|S_r|\to\infty} = \gamma_r\left[|S_r|e^{i\phi} + a\,e^{i\pi p}\,|S_r|^p\,e^{i\phi p}\right] = \gamma_r|S_r|\Bigg[\cos\phi + \underbrace{i\sin\phi + ia\frac{\sin p(\phi-\pi)}{|S_r|^{(1-p)}}}_{\mathrm{Im}\,\alpha=0} + \dots\Bigg]$$

$$\Rightarrow \quad \phi = a\frac{\sin\pi p}{|S_r|^{(1-p)}} \quad \longrightarrow \quad 0^- \text{ for } a < 0$$

$$\Rightarrow \quad \frac{\Gamma_r}{M_r} = -2\,tg\frac{\phi}{2} \approx -\phi \approx |a|\frac{\sin\pi p}{|S_r|^{(1-p)}} \qquad \text{and} \qquad \Rightarrow \quad a_{free} = -\frac{2\sqrt{2\ln 2}\,\gamma}{\sqrt{B}\,\sin\frac{3}{4}\pi}$$

Regge trajectory of free QGP bags

This conclusion is justified and supported by several models:

by Veneziano model, by open/close string model and

by AdS CFT QCD

But FWM also determines the width of large/heavy QGP

exactly corresponds to FWM free bags with $\boldsymbol{\Gamma_r = 2\sqrt{2\ln 2}\,\Gamma_1(m)}$ **and** $\boldsymbol{|S_r| \approx m^2}$ **:**

$$\text{From } \alpha(S_r)\Big|_{|S_r|\to\infty} = \gamma_r\left[|S_r|e^{i\phi} + a\,e^{i\pi p}\,|S_r|^p\,e^{i\phi p}\right] = \gamma_r|S_r|\left[\cos\phi + \underbrace{i\sin\phi + ia\,\frac{\sin p(\phi-\pi)}{|S_r|^{(1-p)}}}_{\mathbf{Im}\,\alpha=\mathbf{0}} + \ldots\right]$$

$$\Rightarrow \quad \boldsymbol{\phi = a\frac{\sin\pi p}{|S_r|^{(1-p)}}} \;\longrightarrow\; \mathbf{0}^- \ \text{ for } a < 0$$

$$\Rightarrow \quad \boldsymbol{\frac{\Gamma_r}{M_r} = -2\,tg\frac{\phi}{2} \approx -\phi \approx |a|\frac{\sin\pi p}{|S_r|^{(1-p)}}} \qquad \text{and} \qquad \Rightarrow \quad \boldsymbol{a_{free} = -\frac{2\sqrt{2\ln 2}\,\gamma}{\sqrt{B}\,\sin\frac{3}{4}\pi}}$$

Use the same logic at high T

As was shown for $T \geq \alpha T_H$ with $\alpha < 1$ the most probable mass $\langle m \rangle \equiv Bv + \Gamma^2(v)\beta > 0$

$\Rightarrow$ in this case $\overline{m}(v) \approx \langle m \rangle$ and $\Gamma_1(v) = \Gamma_1(\langle m \rangle) = \gamma\sqrt{\frac{\langle m \rangle}{B+\gamma^2\beta}} \Rightarrow$

For $$\Gamma_1(\langle m \rangle) \to \infty \quad \Rightarrow \quad \frac{\Gamma_1(\langle m \rangle)}{\langle m \rangle} = \frac{\gamma}{\sqrt{(B+\gamma^2\beta)\langle m \rangle}} \to 0$$

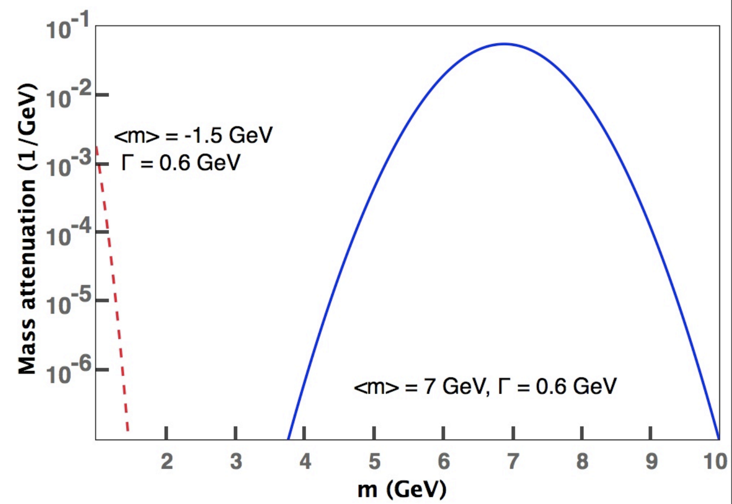
This exactly corresponds to the upper limit of the Regge trajectory behavior $\nu = 1$!

$\alpha(S_r) = \gamma_r[S_r + a(-S_r)^p]$ with $a < 0$ and $p = \frac{3}{4}$ **like for free bags, but for $\langle m \rangle$!**

exactly corresponds to FWM with $\Gamma_r = 2\sqrt{2\ln 2}\,\Gamma_1(\langle m \rangle)$ and $|S_r| \approx \langle m \rangle^2 \approx \overline{m}^2$:

$$\Rightarrow a = -\frac{2\sqrt{2\ln 2}\,\gamma}{\sqrt{(B+\gamma^2\beta)}\,\sin\frac{3}{4}\pi} = -4\sqrt{\frac{2\,T\,T_H}{2\,T - T_H}\ln 2}$$

$$\Rightarrow \Gamma_r = |a|\sqrt{\frac{\langle m \rangle}{2}} \to \infty \quad \text{for} \quad T \to 0.5\,T_H + 0$$



Might be interesting for A+A collisions at low energies (CBM, NICA)

Use the same logic at high T

This is remarkable result since the medium dependent width of extended QGP bags

obeys the upper limit of the asymptotic behavior obtained for point-like hadrons!

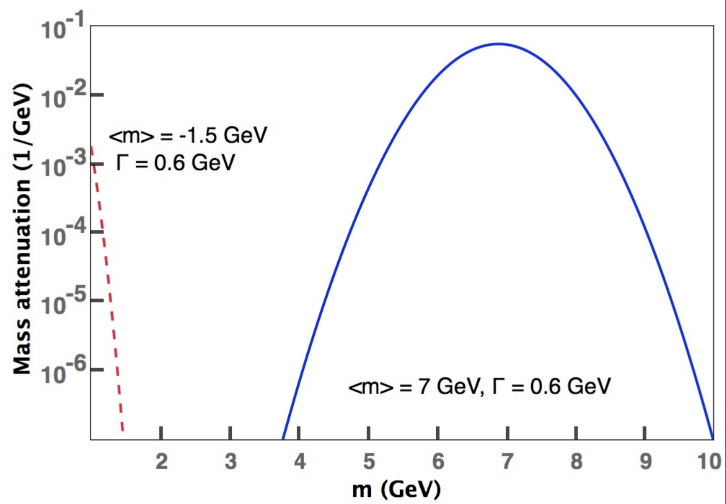
The same result is obtained for other way of averaging!

$$\alpha(S_r) = \gamma_r [S_r + a(-S_r)^p] \quad \text{with} \quad a < 0 \text{ and } p = \frac{3}{4} \quad \text{like for free bags, but for } \langle m \rangle!$$

exactly corresponds to FWM with $\Gamma_r = 2\sqrt{2 \ln 2}\, \Gamma_1(\langle m \rangle)$ and $|S_r| \approx \langle m \rangle^2 \approx \overline{m}^2$:

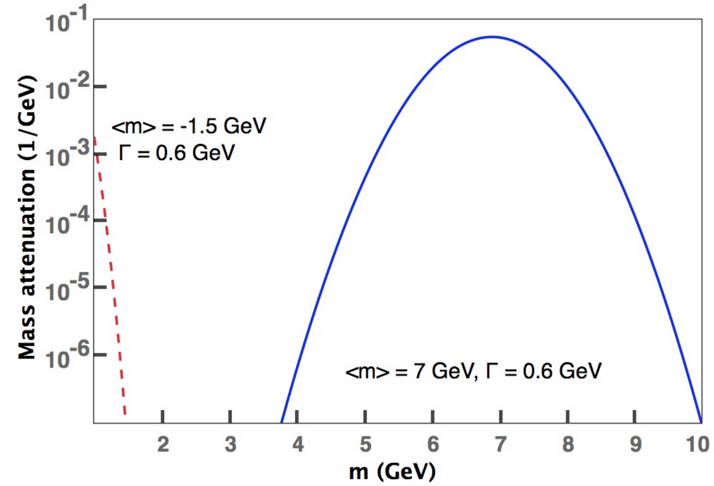
$$\Rightarrow a = -\frac{2\sqrt{2\ln 2}\,\gamma}{\sqrt{(B+\gamma^2\beta)}\,\sin\frac{3}{4}\pi} = -4\sqrt{\frac{2\,T\,T_H}{2\,T - T_H}\ln 2}$$

$$\Rightarrow \Gamma_r = |a|\sqrt{\frac{\langle m \rangle}{2}} \rightarrow \infty \quad \text{for} \quad T \rightarrow 0.5\,T_H + 0$$



Might be interesting for A+A collisions at low energies (CBM, NICA)

Low T case



The case $\boldsymbol{T < \alpha T_H}$ with $\boldsymbol{\alpha < 1}$ opens the new possibility since

the most probable mass $\boldsymbol{\langle m \rangle \equiv Bv + \Gamma^2(v)\beta < 0}$

$\Rightarrow$ in this case $\boldsymbol{\overline{m}(v) \approx M_0 = Const\ of\ v}$ and $\boldsymbol{\Gamma_1(v) = \gamma\sqrt{v}}$ $\Rightarrow$

For $\boldsymbol{\Gamma_1(v) \to \infty}$ $\Rightarrow$ $\boldsymbol{\frac{\Gamma_1(v)}{M_0} = \frac{\gamma\sqrt{v}}{M_0} \to \infty}$

This exactly corresponds to the lower limit of the Regge trajectory behavior $\boldsymbol{\nu = \frac{1}{2}}$!

For $\boldsymbol{S_r \to +\infty}$ Trushevsky's asymptotic $\boldsymbol{\alpha(S_r) = -\gamma_r[-S_r]^{\frac{1}{2}} + C_\alpha}$ with $\boldsymbol{C_\alpha > 0}$

$\boldsymbol{-i\,\gamma_r\,|S_r|^{\frac{1}{2}} + C_\alpha \Leftrightarrow -\frac{i}{2}2\sqrt{2\ln 2}\,\Gamma_1(v) + M_0}$ $\Rightarrow$ **is identical to FWM case $\boldsymbol{\langle m \rangle < 0}$!**

$$\boldsymbol{\Gamma_r \approx 2\,|S_r|^{\frac{1}{2}}, \quad C_\alpha = \gamma_r\,M_0 \quad \text{and} \quad v = \frac{1}{\gamma^2\,2\ln 2}|S_r|}$$

Conclusions

The finite medium dependent width is introduced into statistical model of QGP bags. The model has the Hagedorn-like mass spectrum of heavy bags, but their large width make them hard to observe. This explains a huge deficit in the number of heavy hadronic resonances (resolves the second problem).

For high and low T we derived the general form of the QGP bag's pressure. For low T the model predicts the existence of subthreshold suppression of QGP bags since their most probable mass becomes negative.

This resolves the main conceptual problem and explains the reason why the QGP bags and strangelets cannot be observed at low T even as metastable states.

The model allows one to estimate width of QGP bags from LQCD and find it rather large which makes them hard be observed.

The model establishes the Regge trajectories of large QGP bags, shows that these trajectories have statistical nature. For the first time we have a model in which QGP bags correspond to the linear trajectory at high T and the square root one at low T.

Thanks for your attention!

In a Finite System, however...

• Volume and mass integrals in $F(s,T)$ acquire **finite upper limits**: V_{max} and M_{max}

$\Longrightarrow$ For finite v the statistical suppression of QGP bags in hadronic phase is not about Avogadro number, but a few orders of magnitude!

$\Longrightarrow$ finite v QGP bags can exist in hadronic phase as metastable states!

• **Finite volume solution** of GBM, **K.A.B., Phys. Part. Nucl. 38 (2007)**, allows one

To show that for small V the finite QGP bags are **not suppressed anymore!**

To estimate a decay (formation) time of metastable states $(n = 1, 2, 3, ...)$

$$\tau_n \approx \frac{V_{max}}{\pi n\, V_0\, T} \approx \frac{60\, V_{max}[\mathrm{fm}\cdot\mathrm{MeV}]}{n V_0\, T[\mathrm{MeV}]}$$

Typical example: strangelets

$\Longrightarrow$ at low $T \ll T_c$ the $n \geq 1$ states with **finite QGP bags could exist very long time!**

$\Longrightarrow$ **for small V the finite QGP bags could coexist with hadrons!**

• Moreover, since initial stage of collision is not equilibrated $\Longrightarrow$ **nothing can prevent** the formation of metastable QGP bags in the hadronic phase!

$\Longrightarrow$ **Since the QGP bags were not observed at $T < T_c$, there must exist a reason for that!**