# Tsallis Distribution in High-Energy Heavy Ion Collisions

## an Introduction and More...

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in collaboration

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FAIR-School, 3rd-4th December 2010, Moscow, Russia

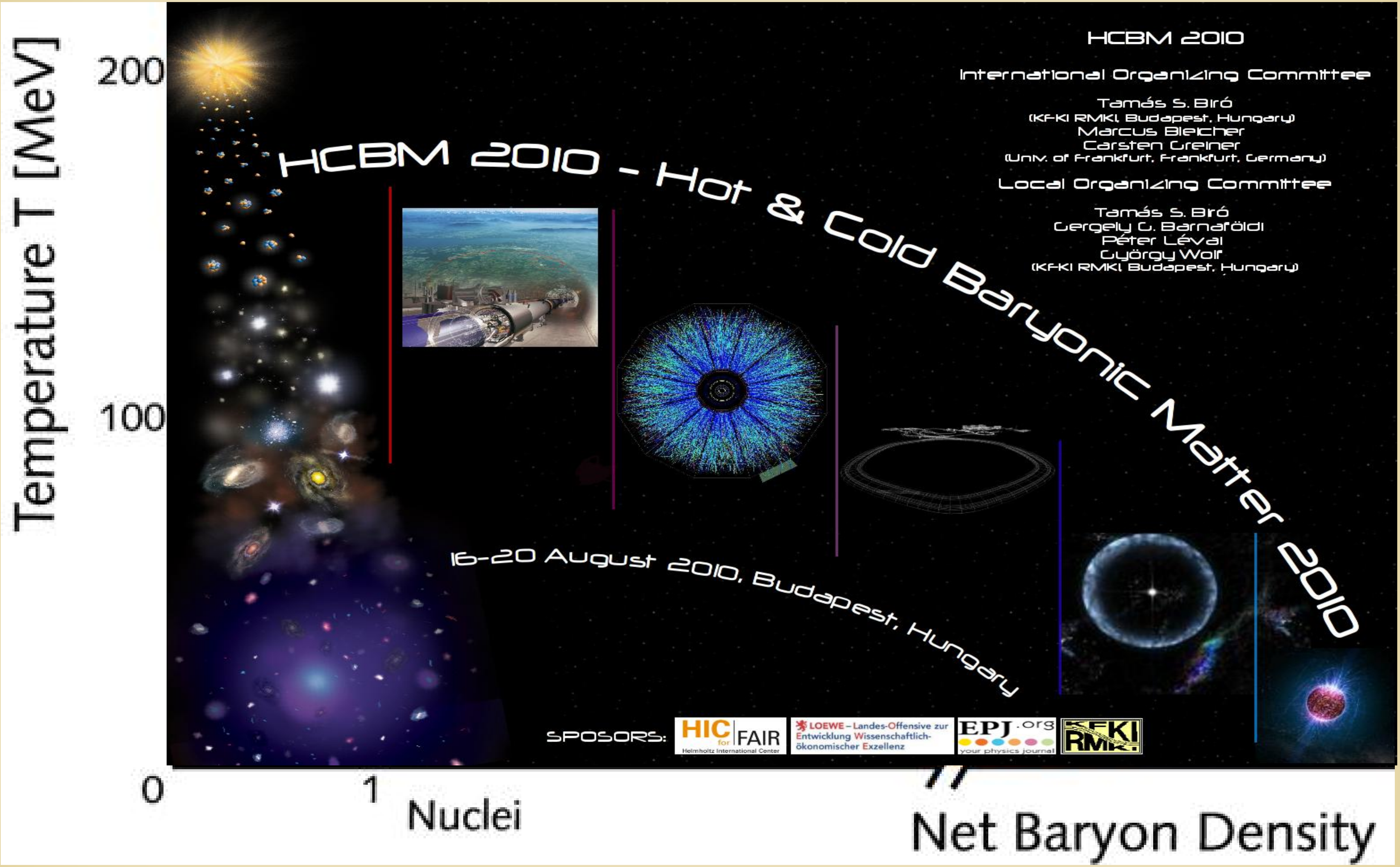
# O U T L I N E

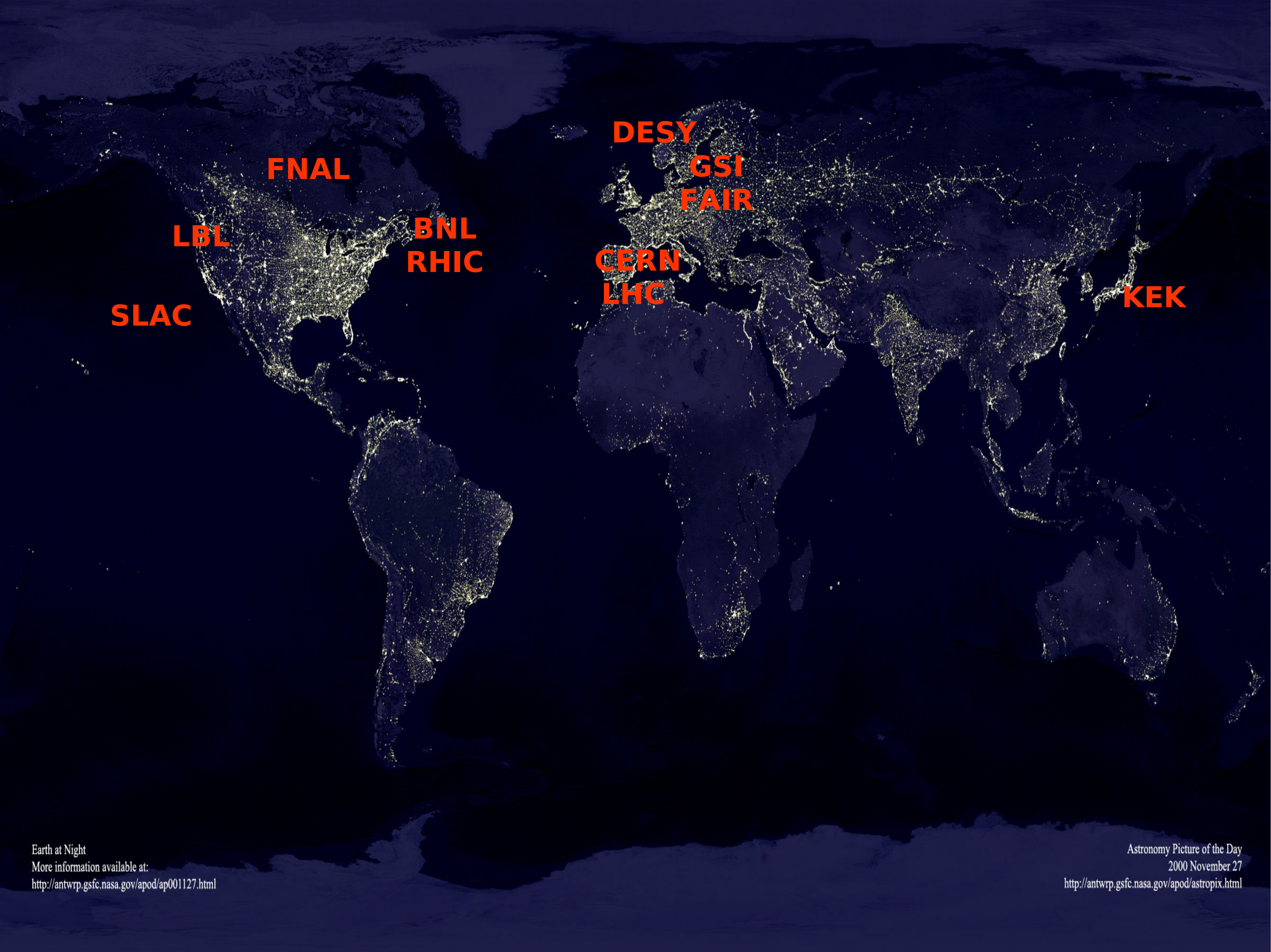
- Introduction & Motivation
- Boltzmann-Gibbs thermodynamics
- Who/Why/What Pareto, Rényi, & Tsallis?
- High-$p_T$ & low-$p_T$ hadron spectra
- What can we learn from HIC?

# Introduction & Motivation:

# Heavy Ion Collisions

# Phase Diagram of Strongly Interacting Matter




DESY
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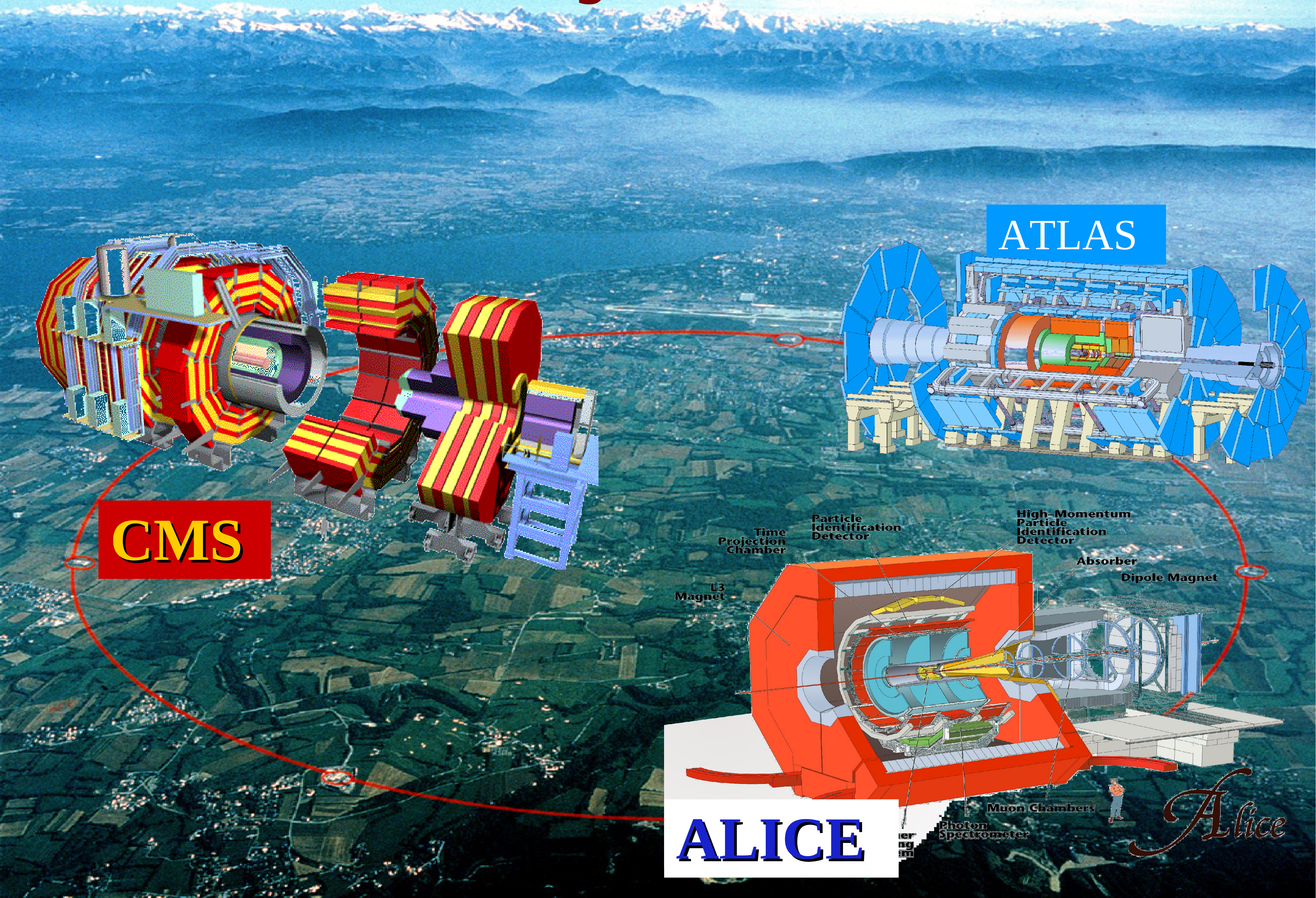
CERN
LHC
KEK
SLAC
Earth at Night
More information available at:
http://antwrp.gsfc.nasa.gov/apod/ap001127.html
Astronomy Picture of the Day
2000 November 27
http://antwrp.gsfc.nasa.gov/apod/astropix.html

# LHC – the Large Hadron Collider

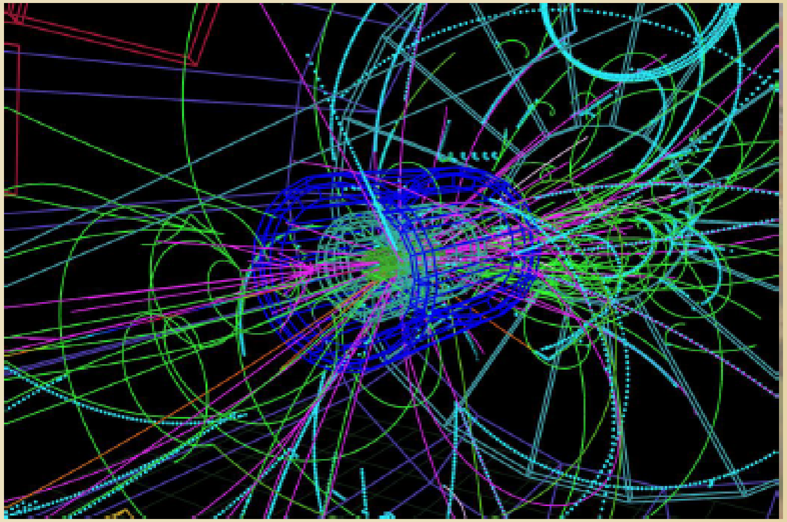


# Heavy Ion Collisions at CERN LHC

- Since 2009 autumn

  Proton-Proton Collisions at 7 TeV

  Particle physics tests: jet physics, Standard Model, Higgs search, baseline for Heavy Ion Experiment

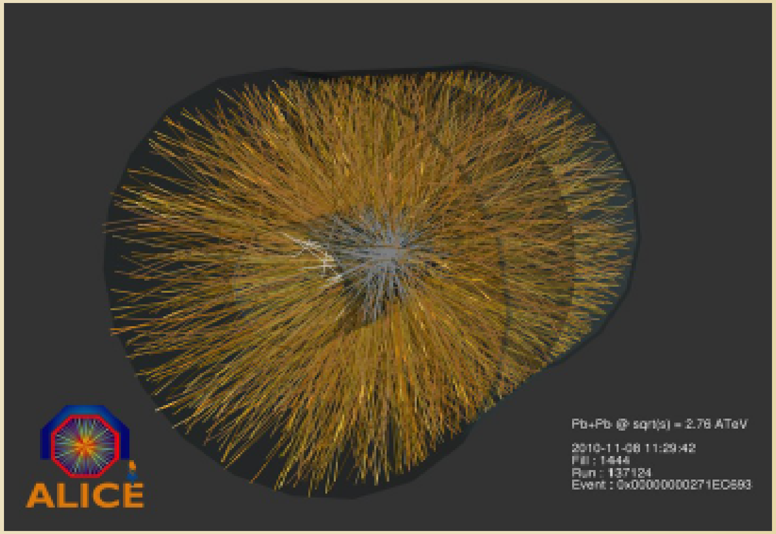


- Now, as of 2010 winter

  Pb-Pb Collisions at 2.76 ATeV

  The real Heavy Ion Collisions

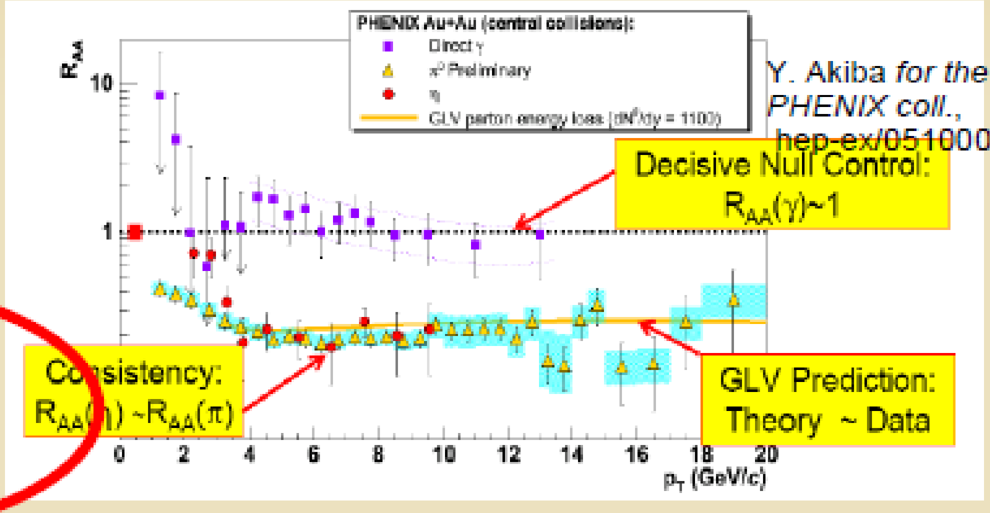
  Test: Quark Gluon Plasma (QGP)

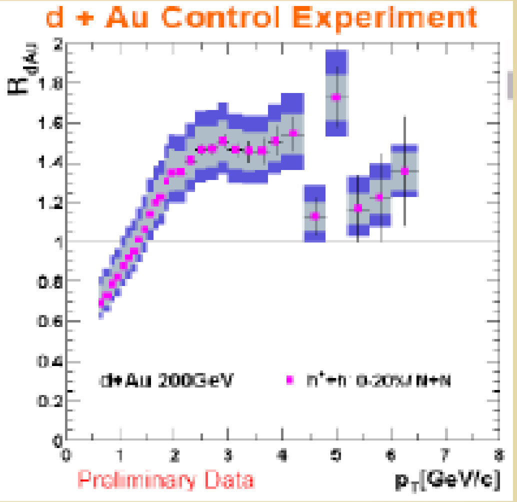


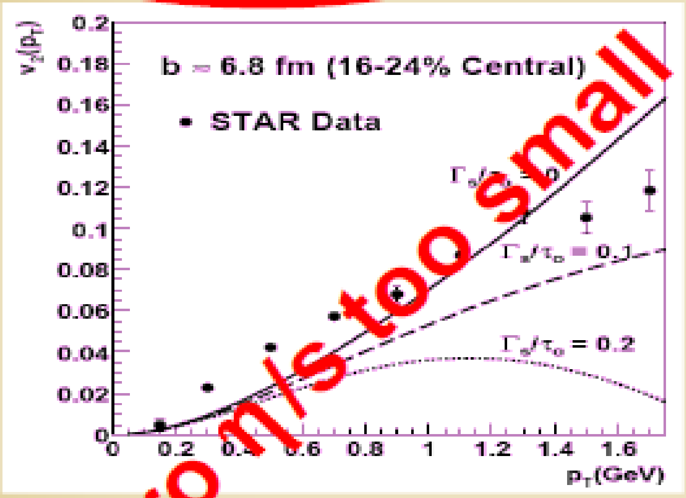
# Measured properites of QGP @ RHIC

**Properties**:

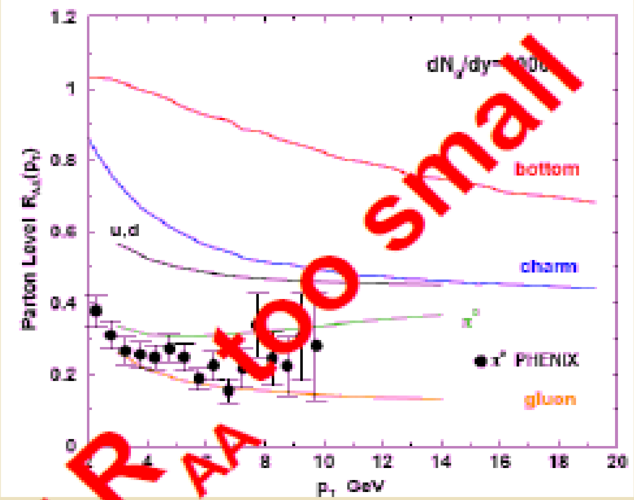
- $dN/dy > 1000$
- $\varepsilon = 1\text{-}10\,\mathrm{GeV/fm^3}$
- $L / \lambda = 3 - 4$
- $L \approx 5$ fm
- $q = 5\text{-}10\,\mathrm{GeV^2/fm}$



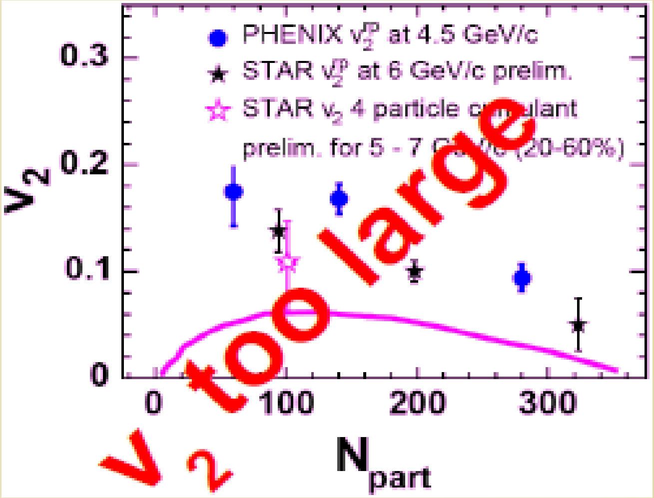




D. Teaney, Phys. Rev. **C68**, 034913 (2003)




M. Djorjevic, M. Gyulassy, R. Vogt, S. Wicks, Phys. Lett. **B632**:81-86 (2006)



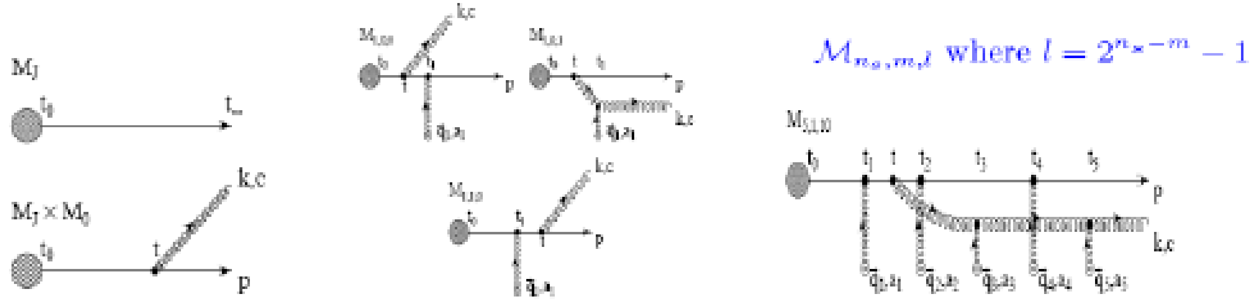

A. Drees, H. Feng, and J. Jia, Phys. Rev. **C71**:034909 (2005) (first by E. Shuryak, Phys. Rev. **C66**:027902 (2002))

G.G. Barnaföldi: QGP vs. AdS/CFT - 2009.

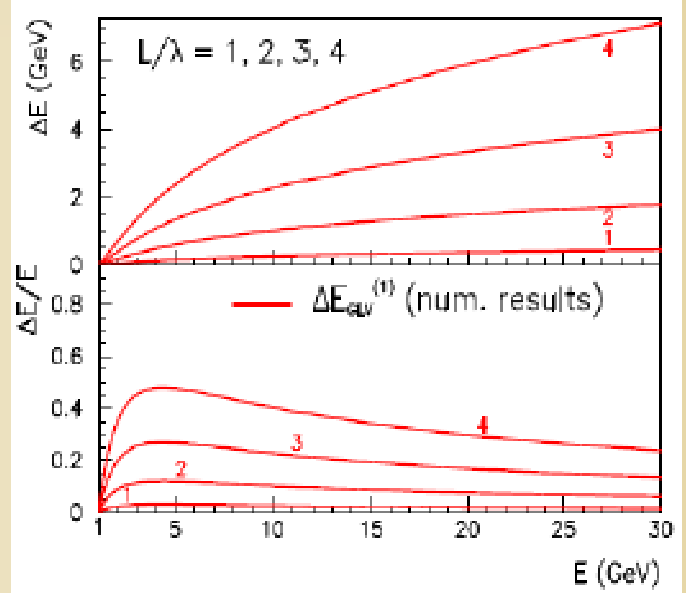
# A signature: Non-Abelian Jet Energy Loss

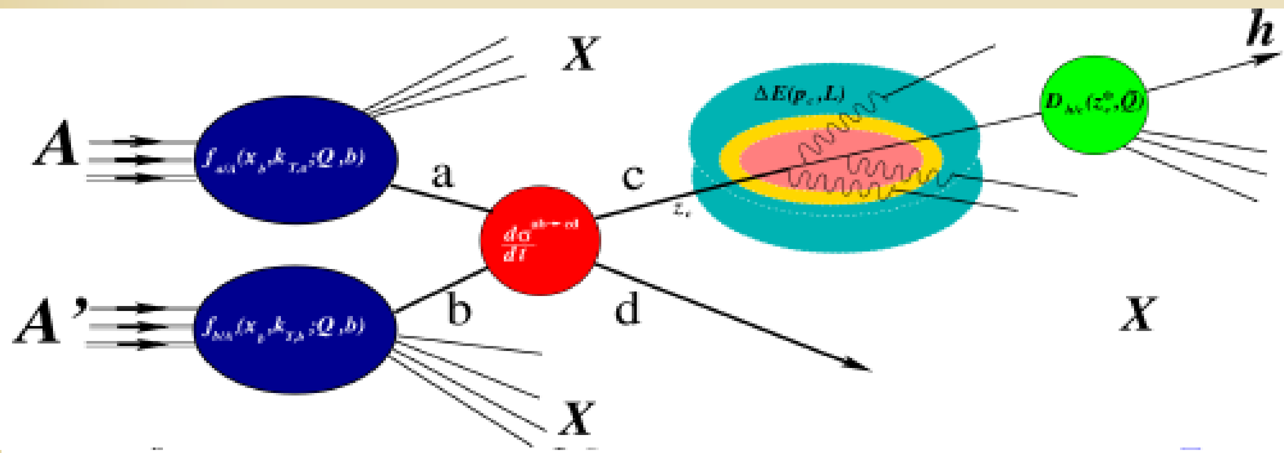
GLV: time-ordered pQCD (Feynman diagramms)
+ OPACITY expansion ($N = 1, 2, 3, \ldots$)
+ kinematical cuts

thin plasma: $L \sim \lambda_g$

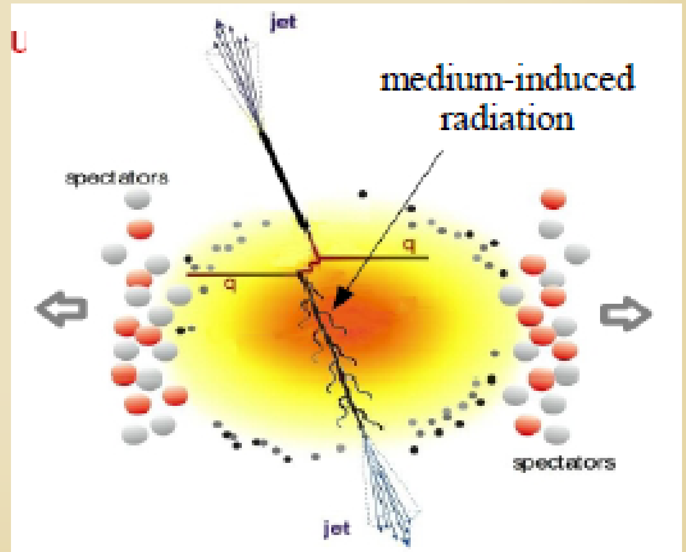



$$\Delta E_{GLV} \approx \frac{C_R \alpha_s}{N(E)} \frac{L^2 \mu^2}{\lambda_g} \log \frac{E}{\mu} = \frac{C_R \alpha_s}{N(E)} \frac{1}{A_\perp} \frac{dN}{dy} \langle L \rangle \log \frac{E}{\langle \mu \rangle}$$

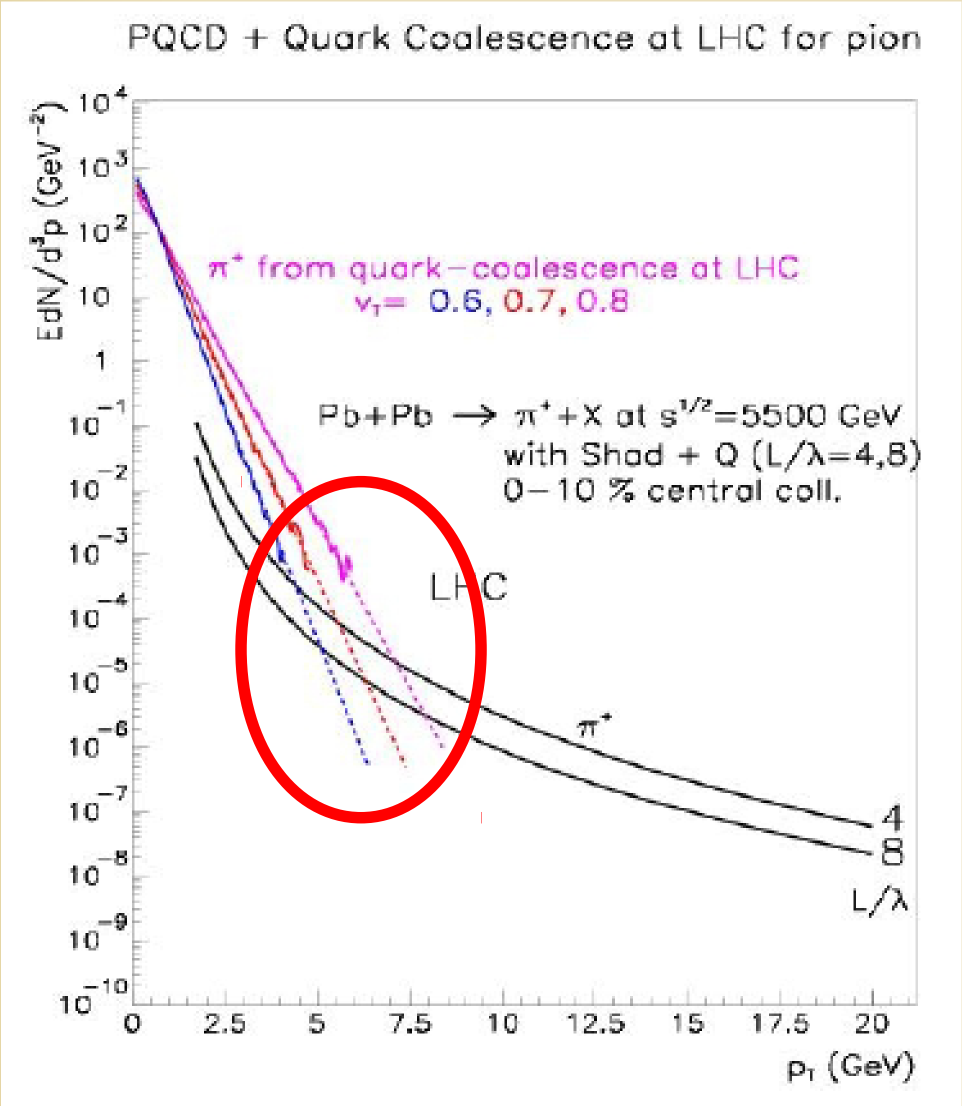



$$\frac{D_{\pi/c}(z_c, Q'^2)}{\pi z_c^2} \longrightarrow \frac{z_c^*}{z_c} \frac{D_{\pi/c}(z_c^*, Q'^2)}{\pi z_c^2}, \quad \text{where} \quad z_c^* = \frac{z_c}{1 - \Delta E / p_c}$$



# Theoretical descriptions of spectra




- pQCD based parton model:

  QCD at T → 0 temperature

  power law distribution

  strong dependence on FF

  good for high-$p_T$ hadrons

- Quark-coalescence model

  Thermal, finite temperature

  exponential distribution $e^{-m / T}$

  parton-hadron duality

  good for high-$p_T$ hadrons

P. Lévai, GGB, G. Fai: JPG35, 104111 (2008)

# Boltzmann-Gibbs

# thermodynamics

# Basics of extensive thermodynamics

Clausius (1865): Concept of Entropy (S) based on macroscopic basis:

$$dS = dQ/T$$

Boltzmann (1872-1877): expressed S can be taken by probability theory from the microscopic states:

$$S_{BG} = -k \sum_{i=1}^{W} p_i \ln p_i \qquad \sum_{i=1}^{W} p_i = 1$$

Here probabilities $\{p_i\}$ corresponding to the W microscopic configuration.

Gibbs, Neumann, Shannon for equally probabilities: $p_i = 1 / W$ for all $i$

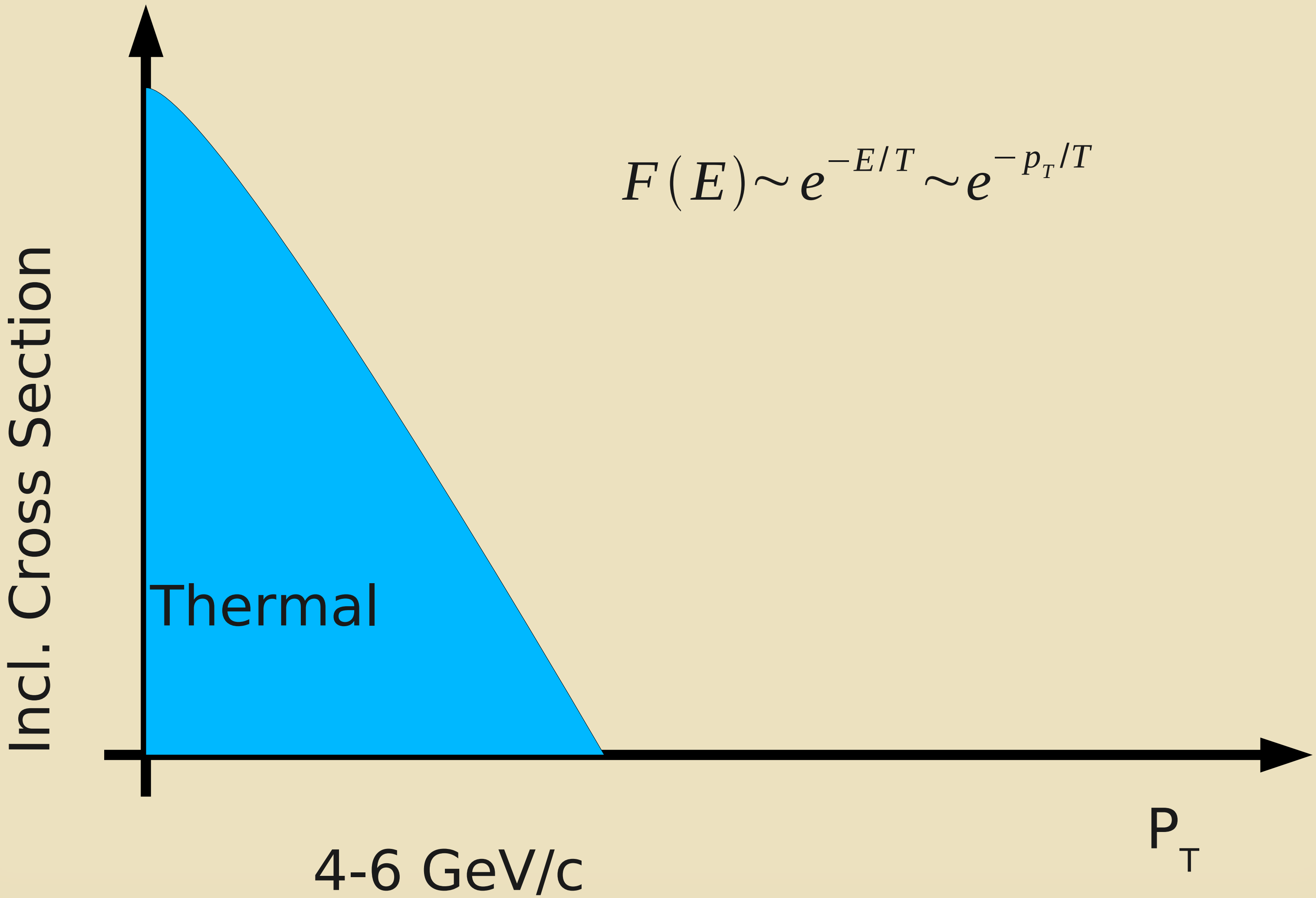
$$S_{BG} = k \ln W$$ with additivity rule: $$S(A+B) = S(A) + S(B)$$

Assuming $N$ equal and independent elements (e.g. to see extensivity):

$$S_{BG}(N) = N \cdot S_{BG}(1) \quad \text{with} \quad \lim_{N\to\infty} S_{BG}(N)/N = S_{BG}(1) < k \ln[W(1)] < \infty$$

# Example: Thermal hadron spectra

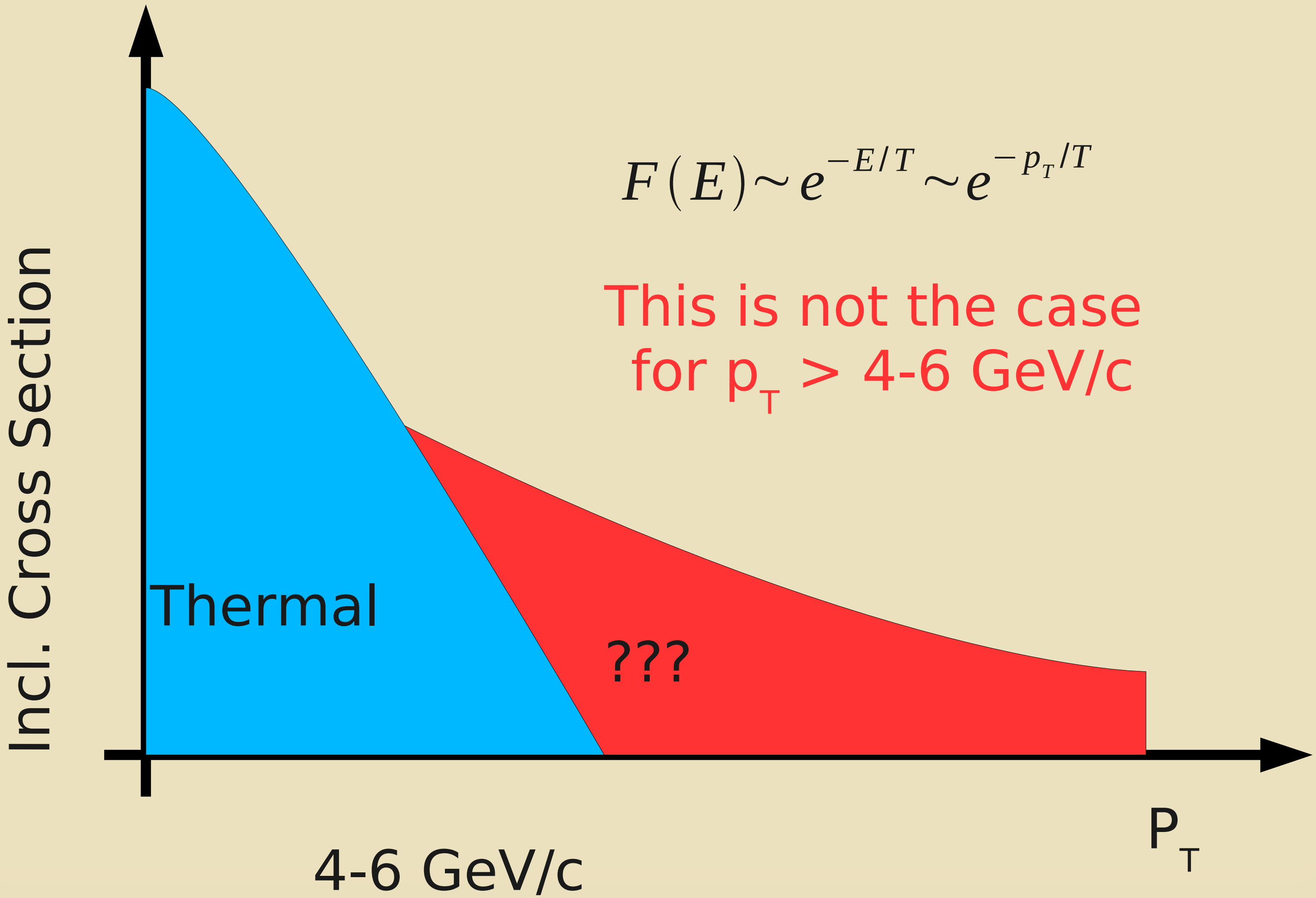
- Thermalised system: spectra follow Boltzmann-Gibbs




$$F(E) \sim e^{-E/T} \sim e^{-p_T/T}$$

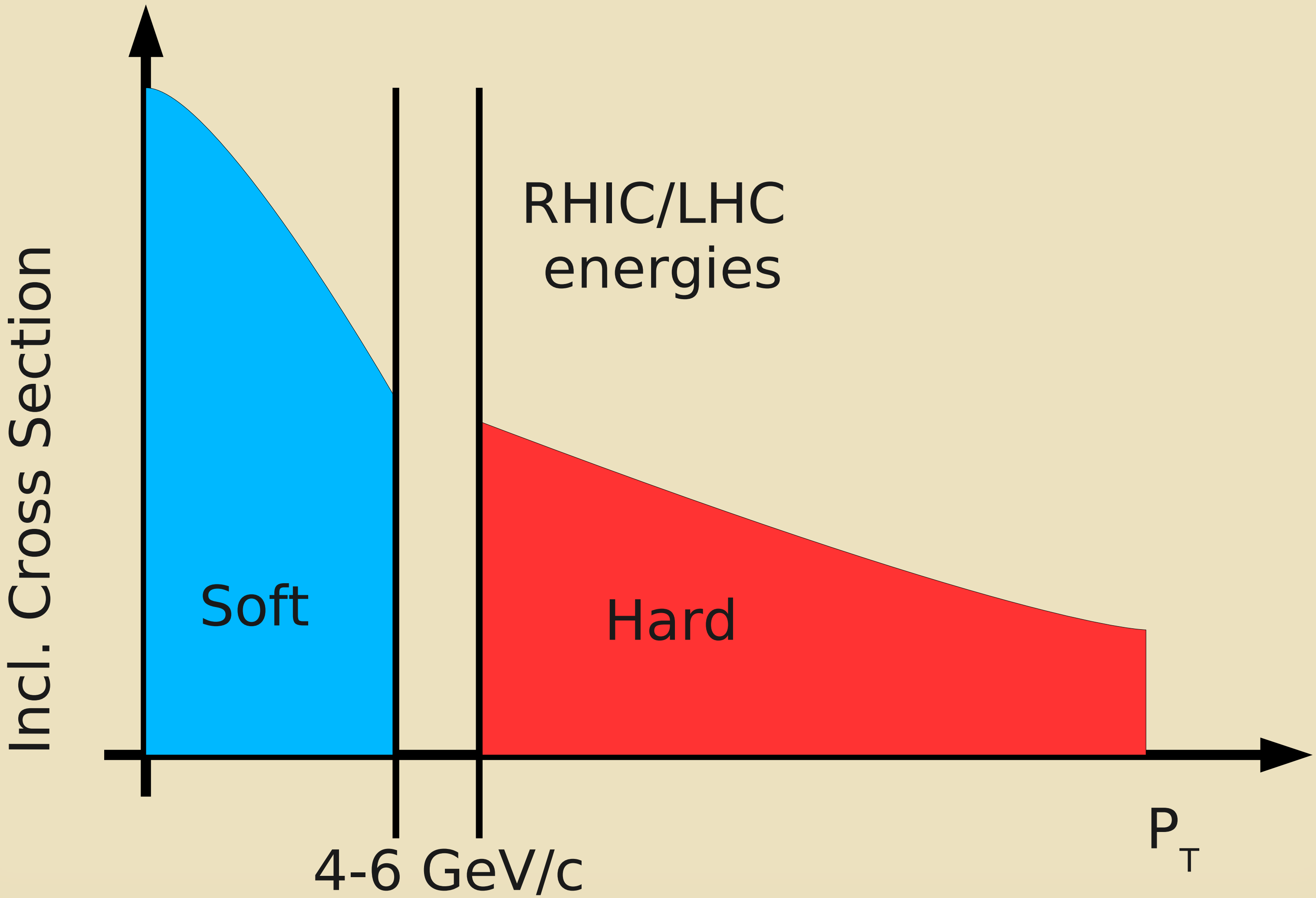
# Example: Thermal hadron spectra

- But in HIC this is quite different...



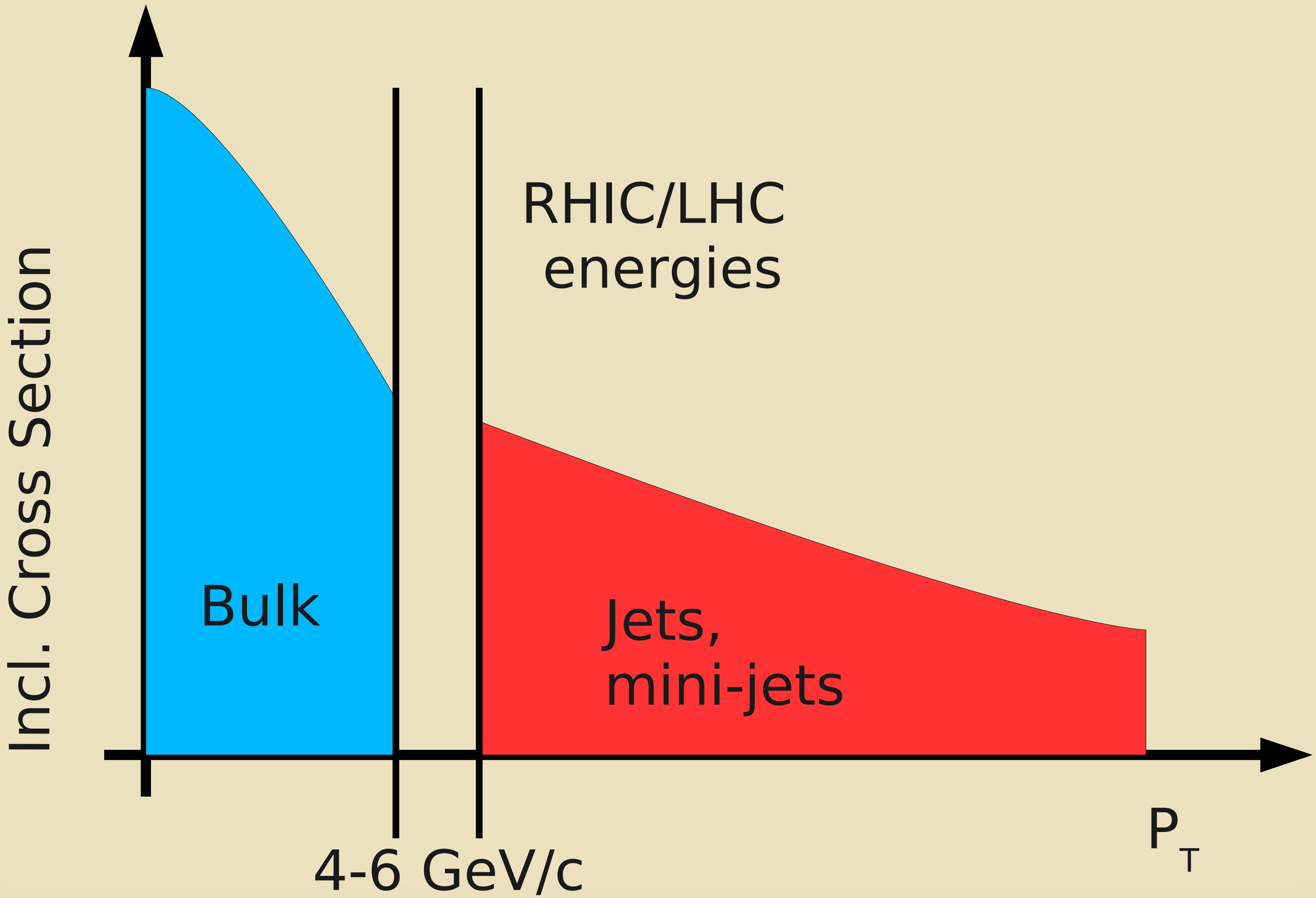
# Models for high & low $p_T$ hadron spectra

- 1. interpretation 'soft' & 'hard' processes



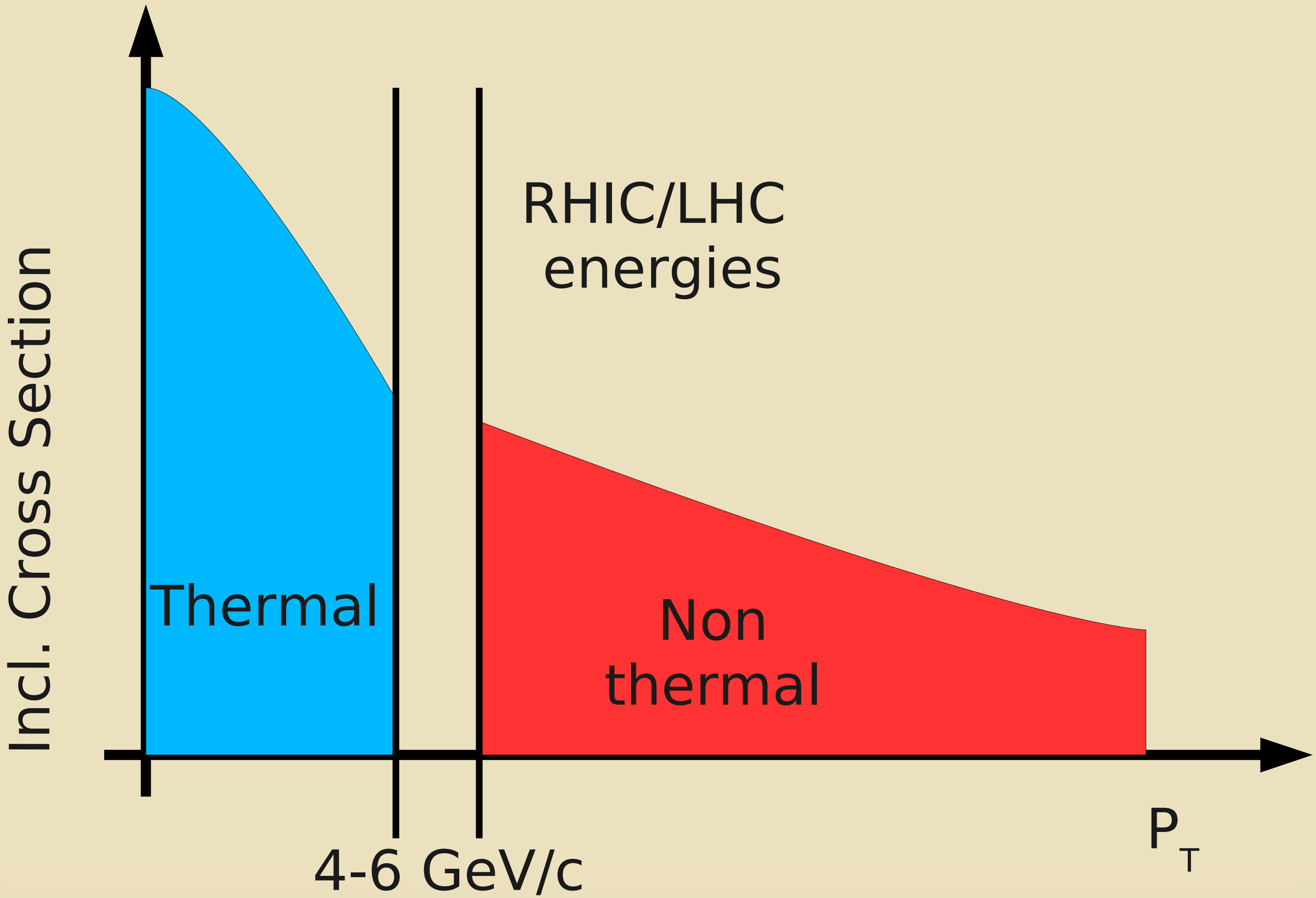
# Models for high & low $p_T$ hadron spectra

- 2. interpretation 'bulk' & 'jet-like' processes



# Models for high & low $p_T$ hadron spectra

- 3. interpretation 'thermal' & 'non-thermal' models



# Who/Why/What

# Pareto, Rényi, & Tsallis

# History: Tsallis, Rényi, Pareto...

- Constantino Tsallis

  1943 (Greece) – (Brazil), statistical phyisicist

  Applying Generlised Entropy: $S_q(p) = \frac{1}{q-1}\left(1 - \int (p(x))^q \, dx\right),$

- Alfréd Rényi

  1921 – 1970 (Hungary), mathematician

  Defines a Generalised Enrtopy: $H_\alpha(X) = \frac{1}{1-\alpha}\log\left(\sum_{i=1}^{n} p_i^\alpha\right)$

- Vilfredo Federico Damaso Pareto

  1848 (Paris) – 1923 (Geneva), (Italian)

  industrialist, sociologist, economist, philosopher

  Generalised Pareto Distrib.: $f_{(\xi,\mu,\sigma)}(x) = \frac{1}{\sigma}\left(1 + \frac{\xi(x-\mu)}{\sigma}\right)^{\left(-\frac{1}{\xi}-1\right)}.$

# Basics of non-extensive thermodynamics

Non-extensive thermodynamics (Based on: T.S. Biró: EPL84, 56003,2008) associative composition rule, (non-additive) :

$$h(h(x,y),z) = h(x,h(y,z))$$

Then should exist a strict monotonic function, X(x) 'generalised logarithm' (an entropy-like quantity), for which:

$$h(x,y) = X^{-1}(X(x) + X(y)) \qquad X(h(x,y)) = X(x) + X(y).$$

Example: (i) Classical thermodynamics:

$$f(E) = e^{-\beta E}/Z \qquad h(x,y) = x + y.$$

(ii) Tsallis distribution

$$h(x,y) = x + y + axy \qquad a = q - 1$$

$$f(E) = \frac{1}{Z} e^{-\frac{\beta}{a}\ln(1+aE)} = \frac{1}{Z}(1+aE)^{-\beta/a} \qquad S = \int f \frac{e^{-a\ln(f)} - 1}{a} = \frac{1}{a}\int (f^{1-a} - f).$$

# Extensive vs. non-extensive

| SYSTEMS | ENTROPY $S_{BG}$ (additive) | ENTROPY $S_q$ ($q$<1) (nonadditive) |
|---|---|---|
| Short-range interactions, weakly entangled blocks, etc | EXTENSIVE | NONEXTENSIVE |
| Long-range interactions (QSS), strongly entangled blocks, etc | NONEXTENSIVE | EXTENSIVE |

quarks-gluons, plasma, curved space ...?

C. Tsallis: EPJ A40 257 (2009)

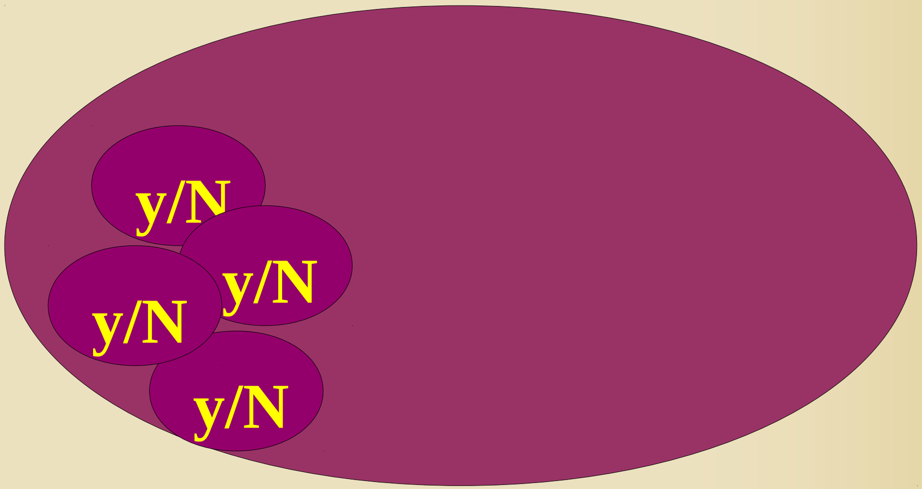
# Associative composition ⇉ evolution eq.

Non-extensive Gibbs, generalised

logarithm: $f(x) = \frac{1}{Z} e^{-\beta X(x)}.$

Composition rule for sub-systems:

$$x_N(y) := \underbrace{h \circ \ldots \circ h}_{N-1} \left( \frac{y}{N}, \ldots, \frac{y}{N} \right)$$




Meanwhile satisfy: $\lim_{N \to \infty} x_N(y) < \infty.$

Assimptotically, if $N_1, N_2 \to \infty$ :

$$x_{N_1+N_2} = \varphi(x_{N_1}, x_{N_2})$$

recursive equation can be given:

$x_n = h\left(x_{n-1}, \frac{y}{N}\right)$, where $h(x,0) = x.$

$$x_n - x_{n-1} = h\left(x_{n-1}, \frac{y}{N}\right) - h(x_{n-1}, 0).$$

Evolution equation can carry out:

$$\frac{dx}{dt} = \frac{y}{t_f} h_2'(x, 0^+)$$

$$L(x) = \int_0^x \frac{dz}{h_2'(z, 0^+)} = y \frac{t}{t_f}.$$

# Some applications of Tsallis distribution

- Laser Cooling

PRL 102, 063001 (2009) PHYSICAL REVIEW LETTERS week ending 13 FEBRUARY 2009

**Power-Law Distributions for a Trapped Ion Interacting with a Classical Buffer Gas**

Ralph G. DeVoe
*Physics Department, Stanford University, Stanford, California 94305, USA*
(Received 3 November 2008; published 10 February 2009)

Classical collisions with an ideal gas generate non-Maxwellian distribution functions for a single ion in a radio frequency ion trap. The distributions have power-law tails whose exponent depends on the ratio of buffer gas to ion mass. This provides a statistical explanation for the previously observed transition from cooling to heating. Monte Carlo results approximate a Tsallis distribution over a wide range of parameters and have *ab initio* agreement with experiment.

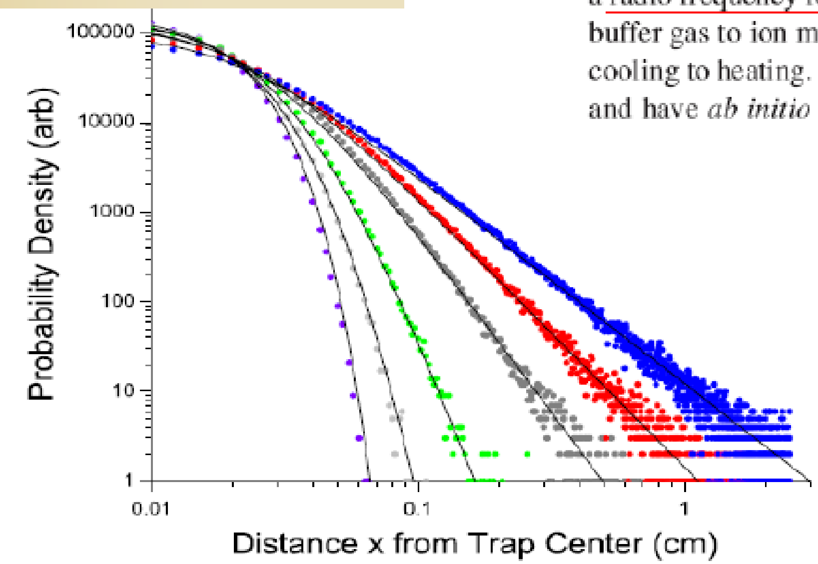



FIG. 1 (color online). Monte Carlo distributions for a single $^{136}Ba^+$ ion cooled by six different buffer gases at 300 K ranging from $m_B = 4$ (left) to $m_B = 200$ (right). Note the evolution from Gaussian to power law (straight line) as the mass increases. The solid lines are Tsallis functions [Eq. (7)] with fixed $\sigma = 0.0185$ cm and the exponents of Table I.

$$T(x) = \frac{T(0)}{\left[1 + (q-1)\left(\frac{x}{\sigma}\right)^2\right]^{\frac{1}{q-1}}}$$

TABLE I. Tsallis parameters $n$ and $q_T$ fit from Fig. 1.

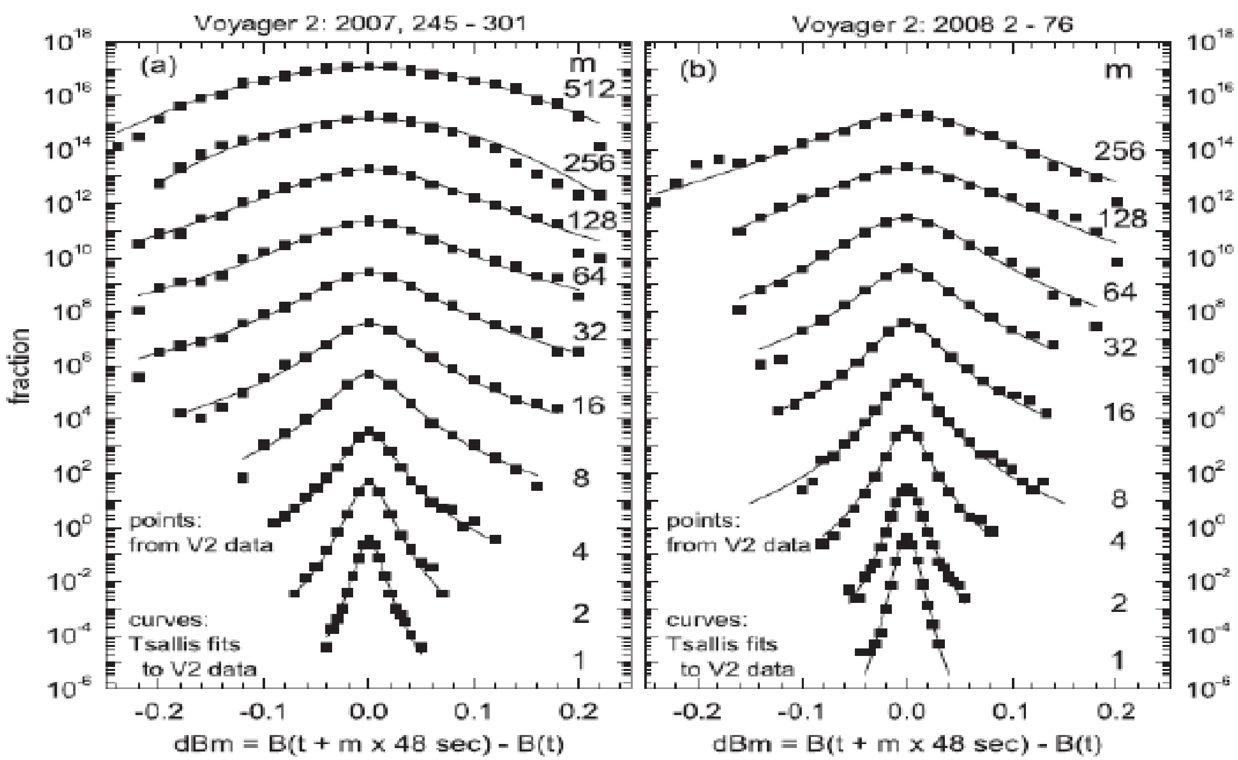
# Some applications of Tsallis distribution

- Voyager2 measured:

THE ASTROPHYSICAL JOURNAL, 703:311–324, 2009 September 20
doi:10.1088/0004-637X/7


COMPRESSIBLE "TURBULENCE" OBSERVED IN THE HELIOSHEATH BY *VOYAGER 2*

L. F. BURLAGA[1] AND N. F. NESS[2]
[1] Geospace Physics Laboratory, Code 673, NASA Goddard Space Flight Center, Greenbelt, MD 20771, USA; Leonard.F.Burlaga@NASA.gov
[2] Institute for Astrophysics and Computational Sciences, Catholic University of America, Washington DC 20064, USA; nfnudel@yahoo.com
*Received 2009 June 2; accepted 2009 July 22; published 2009 August 27*



# Some applications of Tsallis distribution

- Solar neutrinos:

ELSEVIER Physics Letters B 369 (1996) 308–312

## Generalized statistics and solar neutrinos

G. Kaniadakis, A. Lavagno, P. Quarati [1]

*Dipartimento di Fisica and INFM-Politecnico di Torino, Corso Duca degli Abruzzi 24, 10129 Torino, Italy*

*Istituto Nazionale di Fisica Nucleare, Sezioni di Cagliari e di Torino, Italy*

Received 7 November 1995

Editor: R. Gatto

**Abstract**

The generalized Tsallis statistics produces a distribution function appropriate to describe the interior solar plasma, thought as a stellar polytrope, showing a tail depleted with respect to the Maxwell-Boltzmann distribution and reduces to zero at energies greater than about $20 k_B T$. The Tsallis statistics can theoretically support the distribution suggested in the past by Clayton and collaborators, which shows also a depleted tail, to explain the solar neutrino counting rate.

# Some applications of Tsallis distribution

- Hadron jets in electron-positron collision:

I. Bediaga, E.M.F. Curado and J.M. de Miranda, Physica A **286** (2000) 156

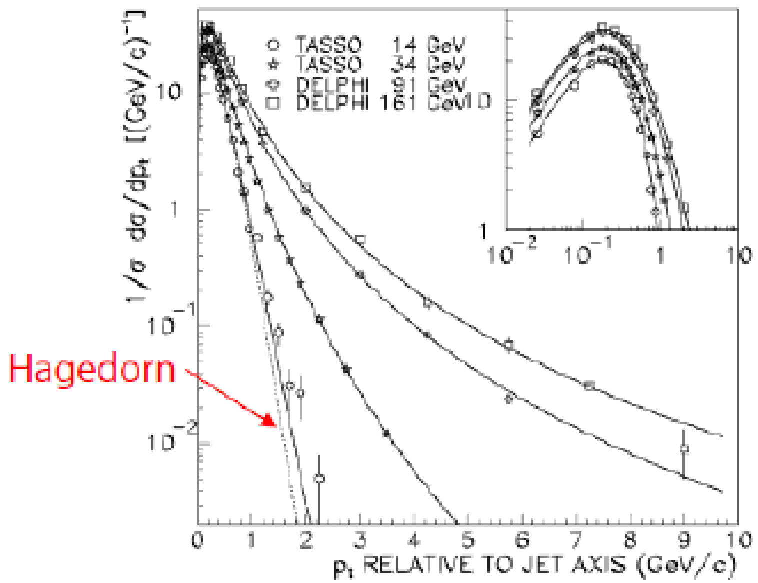
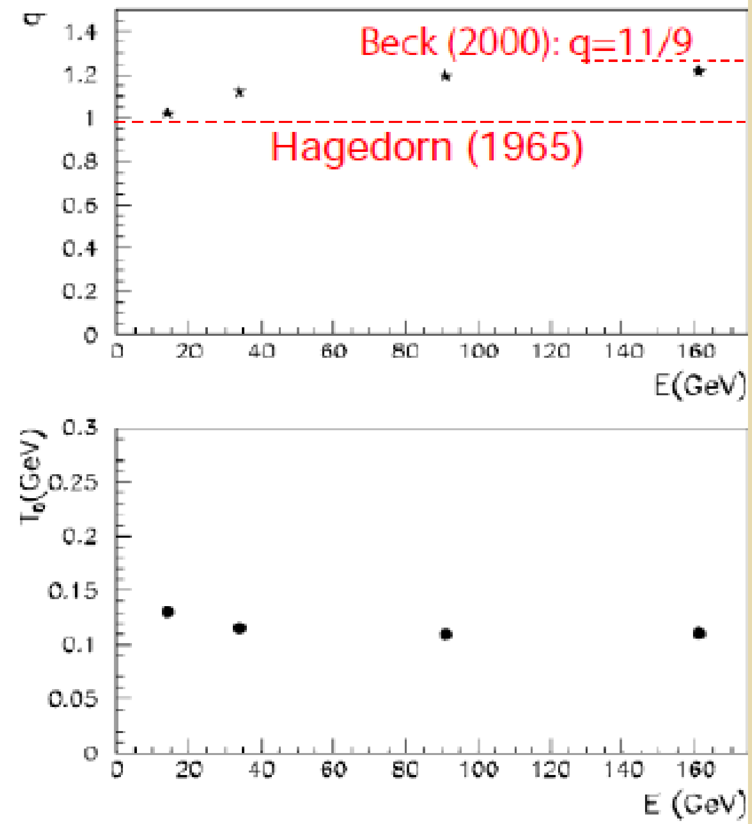



Fig. 1. Transverse momentum distribution. The distribution $(1/\sigma)\,d\sigma/dp_t$ of the transverse momentum $p_t$ of charged hadrons with respect to jet axis (defined in these experimental results as the sphericity axis) is sketched for four different experiments, whose center-of-mass energies vary from 14 and 34 GeV (TASSO) up to 91 and 161 GeV (DELPHI). The Hagedorn predicted exponential behavior is shown by the dotted line. We can see that the deviation of the exponential behavior increases when the energy increases. The continuous lines are obtained from our Eq. (3) and agree very well with the experimental data. The inset shows the transverse momentum distribution for small values of $p_t$.



# Some applications of Tsallis distribution

- Heavy Ion collisions:

Eur. Phys. J. A 40, 325–340 (2009)
DOI 10.1140/epja/i2009-10806-6

THE EUROPEAN PHYSICAL JOURNAL A

Regular Article – Theoretical Physics

## Non-extensive approach to quark matter*

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KFKI Research Institute for Particle and Nuclear Physics, H-1525 Budapest, PF. 49, Hungary

Received: 31 January 2009
Published online: 27 May 2009 – © Società Italiana di Fisica / Springer-Verlag 2009
Communicated by U.-G. Meißner

**Abstract.** We review the idea of generating non-extensive stationary distributions based on abstract composition rules of the subsystem energies, in particular the parton cascade method, using a Boltzmann equation with relativistic kinematics and modified two-body energy composition rules. The thermodynamical behavior of such model systems is investigated. As an application hadronic spectra with power law tails are analyzed in the framework of a quark coalescence model.

# What can we learn

# from Heavy Ion Collisions?

# The first data from the LHC

- New LHC pp data (CMS)

  JHEP 1002:041(2010)

  fitted Tsallis distribution for $p_T$ spectra:

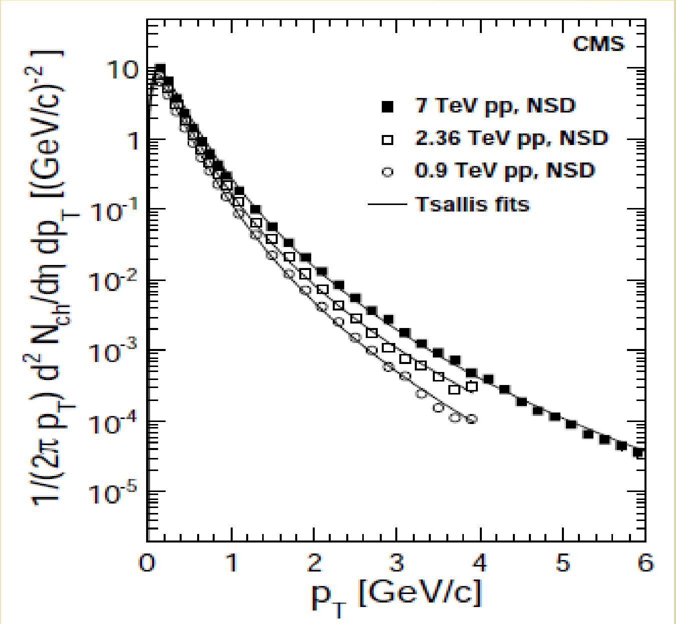
$$E\frac{d^3N_{ch}}{dp^3} = \frac{1}{2\pi p_T}\frac{E}{p}\frac{d^2N_{ch}}{d\eta dp_T} = C(n,T,m)\frac{dN_{ch}}{dy}\left(1+\frac{E_T}{nT}\right)^{-n}$$

  Parameters:

  0.9 TeV T= 130 MeV, q=1.13

  2.36 TeV T= 140 MeV, q=1.15

  $n := (q-1)^{-1}$




- RHIC analysis on AuAu data (y=0)

  Cooper-Frye model: K. Ürmössy, T.S. Bíró: PL B689 14 (2010)

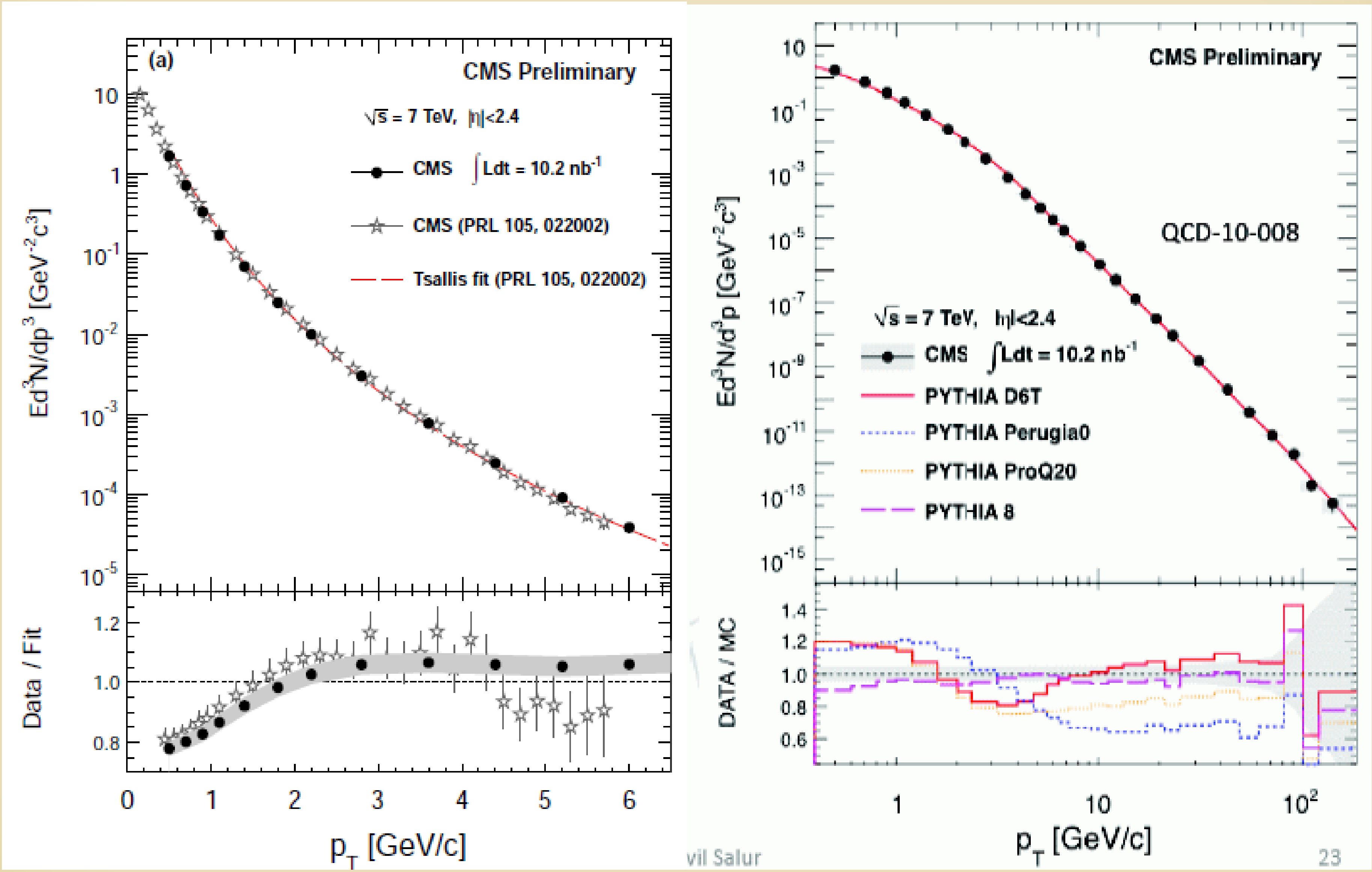
  Parameters:

$$f(E) = A[1+(q-1)E/T]^{-1/(q-1)}$$

  200 GeV T= 51 MeV, q= 1.062 (fit for $p_T$ < 6 GeV/c)

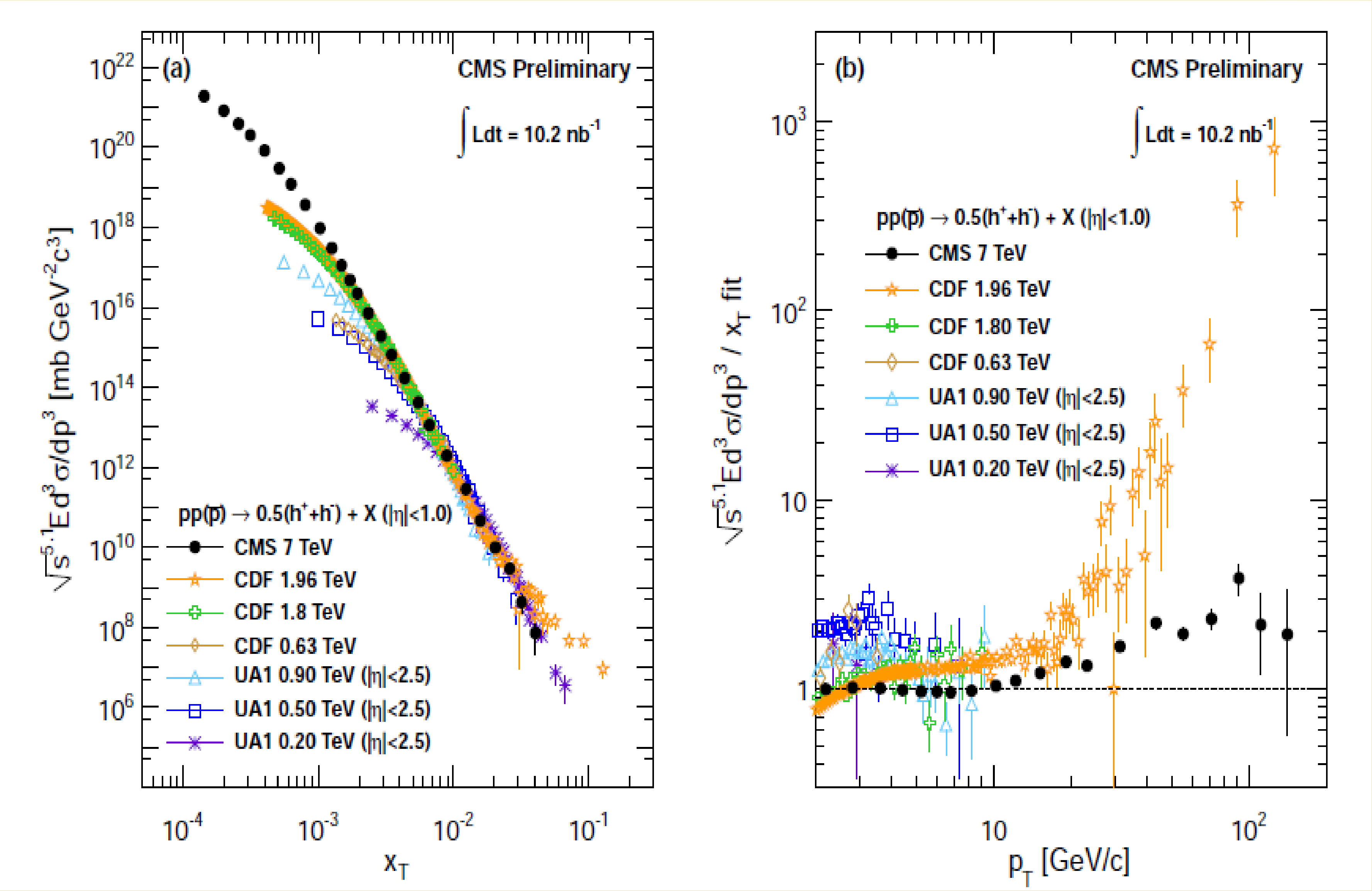
# New data at LHC energies – 2010 summer

See: ALICE: Prague Jet workshop & CMS: QCD-10-008

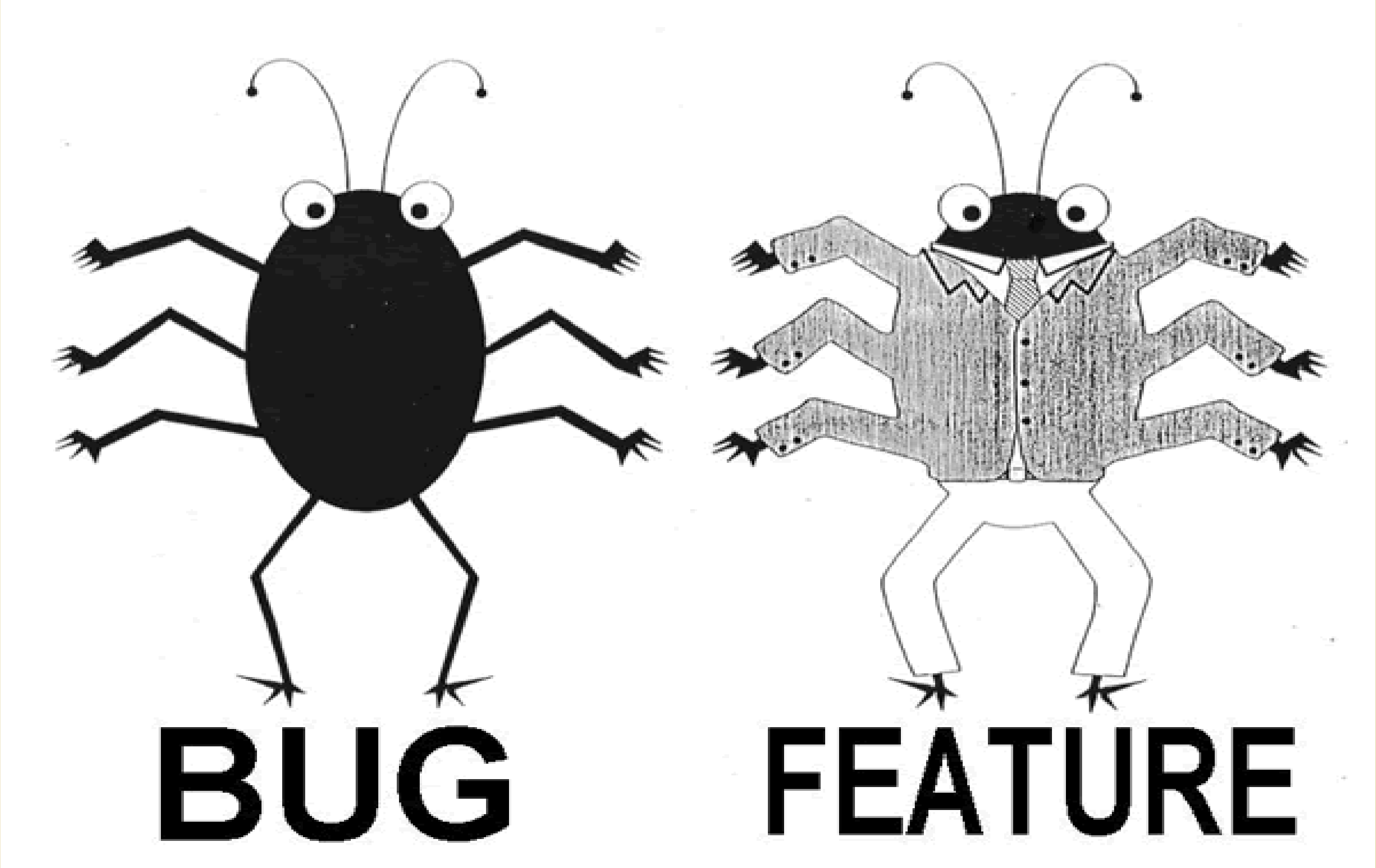


# Comparision in $x_T$: old/new data by Tevatron

See: CDF: PRD 79 112005 (2009) & CMS QCD-10-008



# A new question on the market...

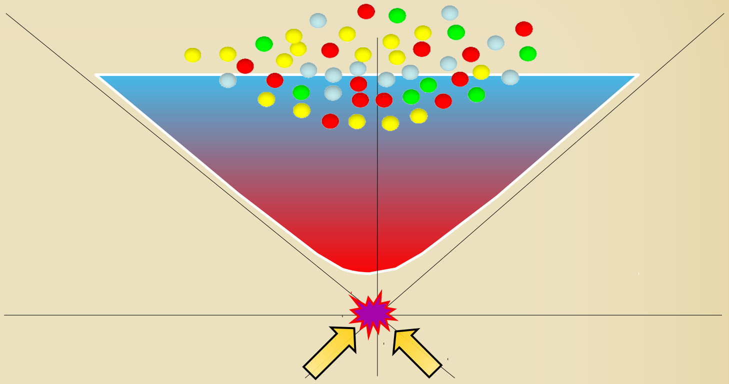


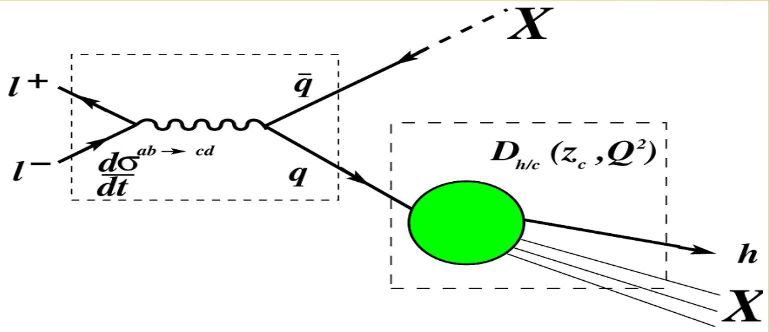
# High-$p_T$ & low-$p_T$

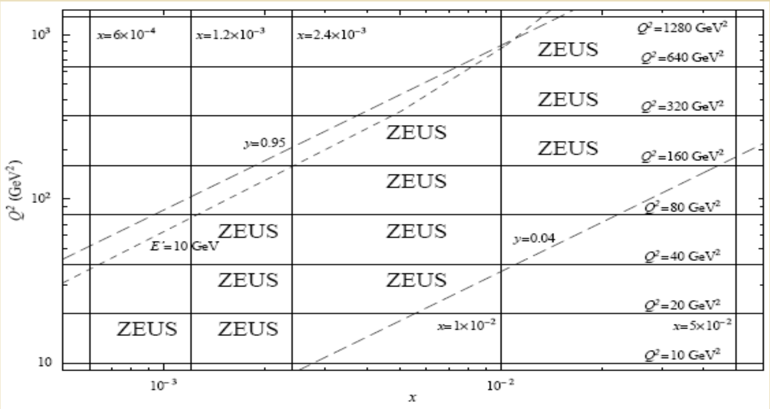
# hadron spectra

# Hadronization processes & fragmentation

- Hadronization: requires a model, based on local parton-hadron duality (kvantum numbers & momenta connected to a cone around or to the leading particle.)
- Parton/hadron shower evolution comes from statistical processes (step-by-step MC evolution). → microscopical
- Fragmentation function (FF) carries integrated (phenomenological) information on how parton fragment into hadron. → integradted distribution
- Measurement lepton-antilepton annihilation, HIC, etc...



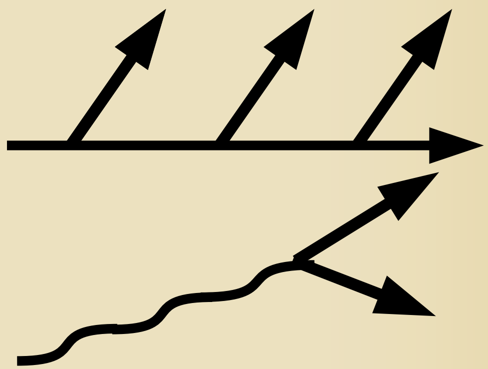


# Models for fragmentation

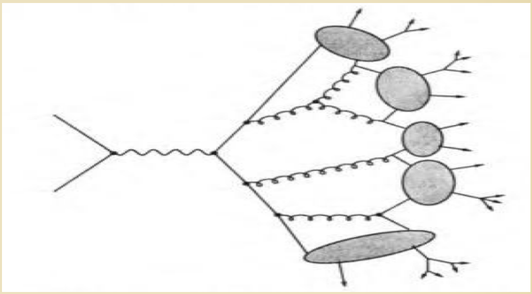
- Independent fragmentation model (Feynman - Field)

  Simplest model for fragmentation by
  Field & Feynman : q & g channels



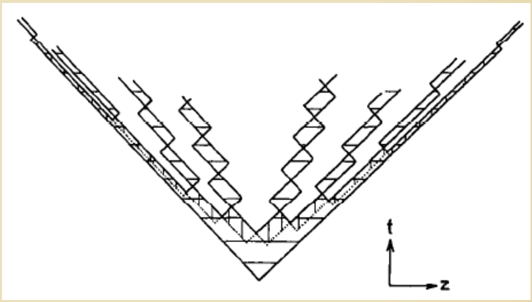
- (Quark) string model

  color strings between $q\bar{q}$, breaking into
  quark-antiquark pair → mesons



- Cluster model (Lund model)

  Lund model: phase-space separation,
  froming clusters: $q\bar{q}$ → M, qqq → B

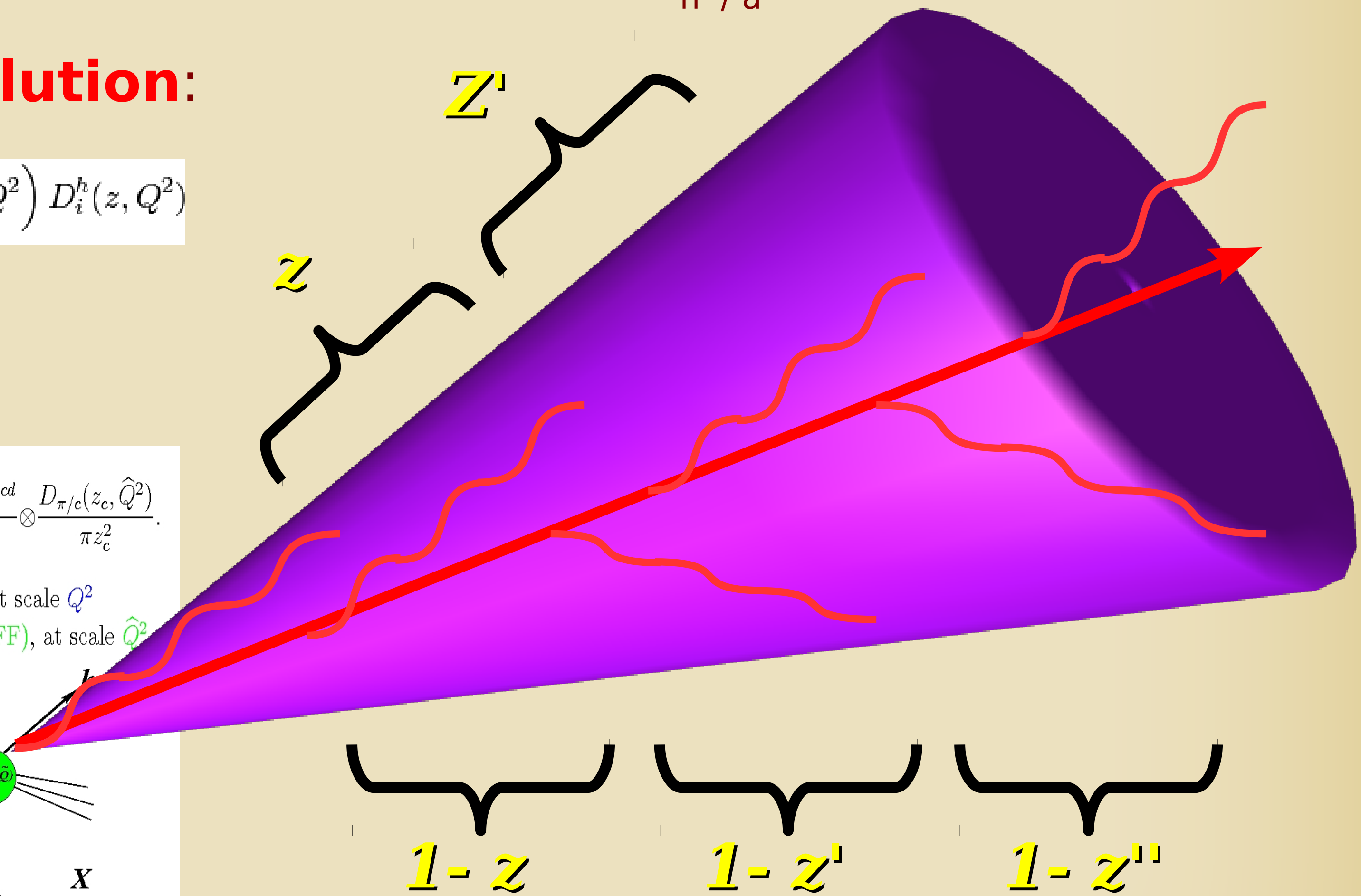


# Fragmentation processes in parton model

In a pQCD based parton model, fragmentation functions (FF) gives how parton (a) fragment into a hadron (h), $D_{h/a}(z,Q^2)$.

**DGLAP scale evolution**:

$$\frac{\partial}{\partial \ln Q^2} D_i^h(x,Q^2) = \sum_j \int_x^1 \frac{dz}{z} \frac{\alpha_S}{4\pi} P_{ji}\left(\frac{x}{z}, Q^2\right) D_i^h(z,Q^2)$$

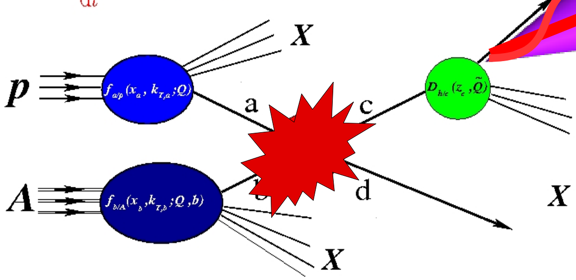



$$E_\pi \frac{d\sigma_\pi^{pA}}{d^3p_\pi} \sim f_{a/p}(x_a,Q^2;k_T) \otimes f_{b/A}(x_b,Q^2;k_T,b) \otimes \frac{d\sigma^{ab\to cd}}{d\hat{t}} \otimes \frac{D_{\pi/c}(z_c,\hat{Q}^2)}{\pi z_c^2}.$$

$f_{b/A}(x_a,Q^2;k_T,b)$ : Parton Dist. Function (PDF), at scale $Q^2$

$D_{\pi/c}(z_c,\hat{Q}^2)$ : Fragmentation Function for $\pi$ (FF), at scale $\hat{Q}^2$

$\frac{d\sigma^{ab\to cd}}{d\hat{t}}$ : Partonic cross section



# Fragmentation via associative composition

Program:

1) Search and fit Tsallis distribution to data.

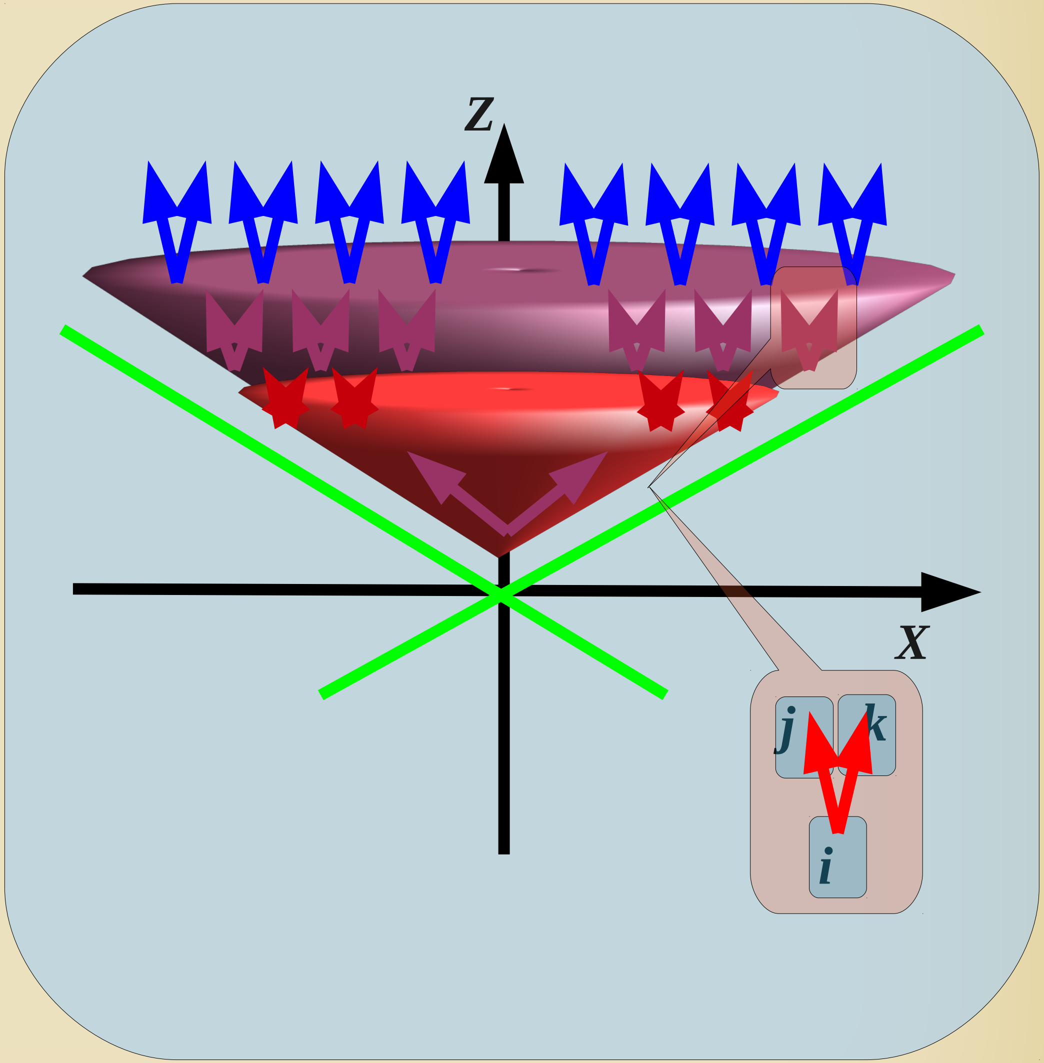
2) Search for physical meaning of T and q parameters.

3) Components of the sub-systems are e.g. 'splitting functions' $P_{q\,g}$, $P_{g\,g}$

4) Test: can a BFKL / DGLAP-like evolution equation be obtained?

$$D(x,Q^2) \sim f(E,T,q) * f(\ln(Q^2))$$

$$D(x,Q^2) \sim f(E,T(\ln(Q^2)),q(\ln(Q^2)))$$



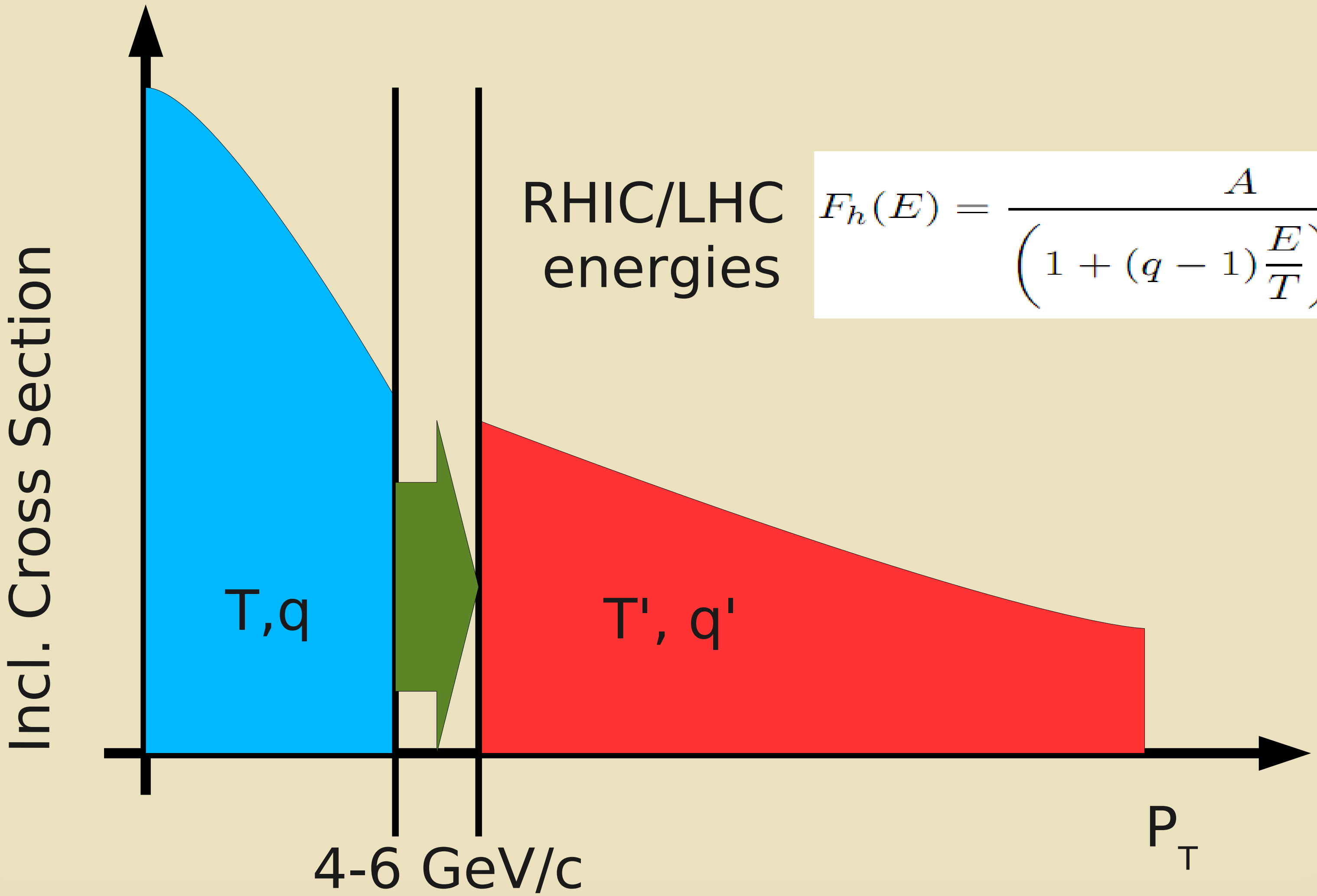
How can Tsallis
(re)solve this?

A test in two models

joint ideas…

# $1^{st}$ way: find the distribution

- A suggested new way: Tsallis (like) distribution




$$F_h(E) = \frac{A}{\left(1 + (q-1)\frac{E}{T}\right)^{1/(q-1)}}$$

# 1st joint model: recombination & pQCD in AA

Find a distribution for low & high momentum spectra:

$$e^{-\beta E} \;\longrightarrow\; F_h(E) = \frac{A}{\left(1+(q-1)\frac{E}{T}\right)^{1/(q-1)}}$$

In AA collisions particle energy modified by the flow:

$$E = \gamma(m_t - v_{flow} p_t), \quad m_t = \sqrt{m^2 + p_t^2}$$

Slope for particle spectra can be fitted:

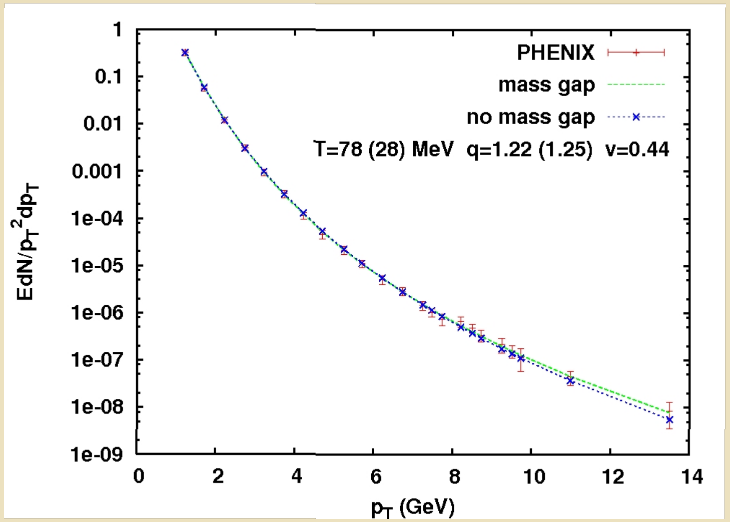
$$T_{slope} = -\frac{\partial E}{\partial ln(F_h(E))} = T(1+aE) = T + (q-1)E$$

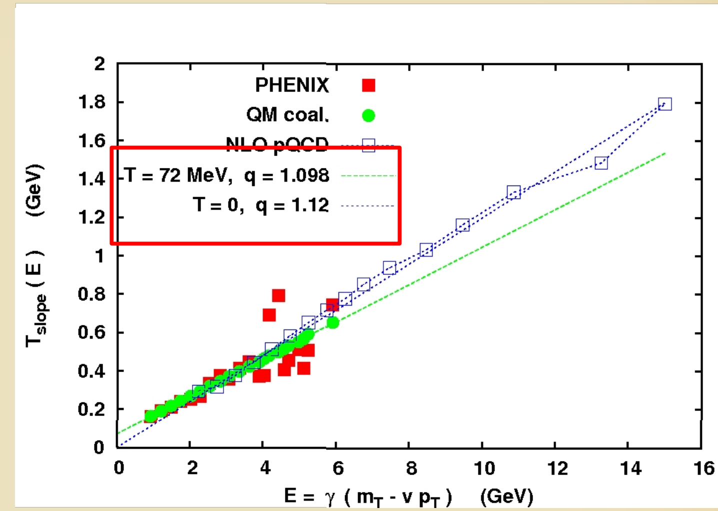
furthermore, if E>>m, then $T \rightarrow T\,[\,(1+v)/(1-v)\,]^{1/2}$

See more on details by: K. Ürmössy and P. Ván

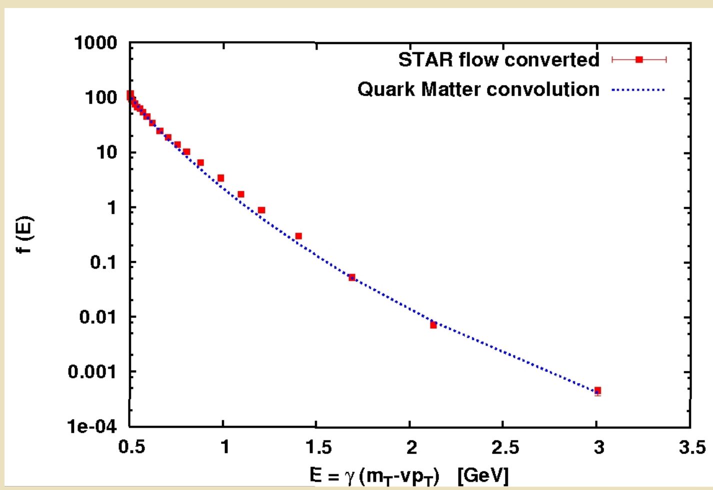
# $1^{st}$ joint model: recombination & pQCD in AA

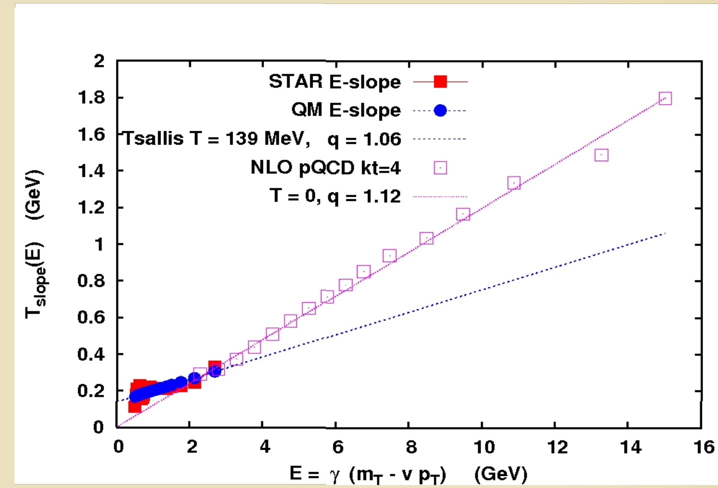
Pion spectrum




Kaon spectrum



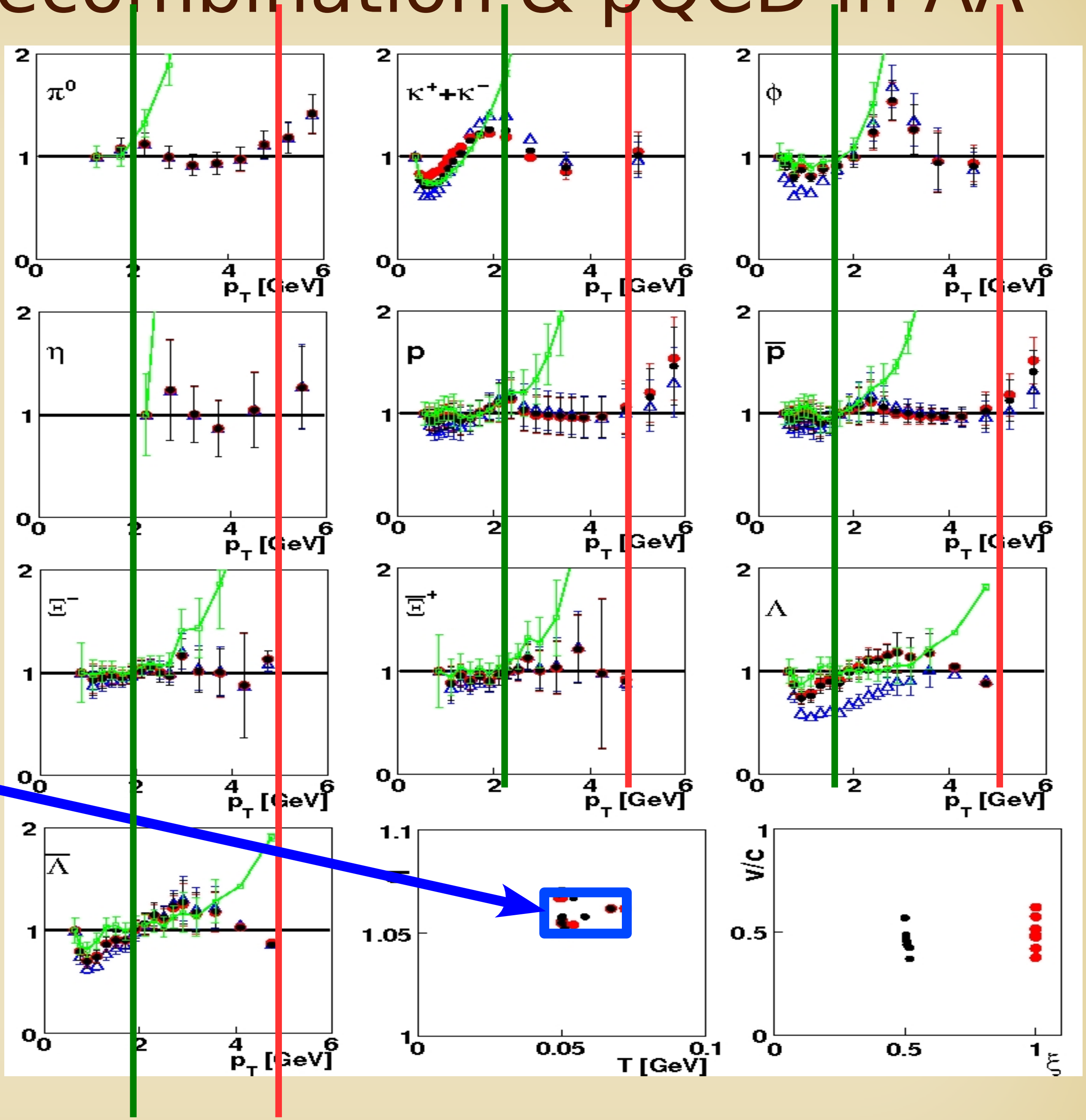




Fitted T & q Tsallis parameters at RHIC energies: (recombination and NLO pQCD+AKK FF, $v=0.5$ for kaons): T.S. Bíró, K. Ürmössy, GGB: JPG35, 044012 (2008)

# 1st joint model: recombination & pQCD in AA

Analysing
Meson & Baryon
specta in
AuAu collisions

- Ratio of theoretical or experimental $p_T$ spectra in y=0, AuAu collisions fitted by Tsallis distribution.
- T=50-70MeV, q=1.06-1.07
- Here the fit is only for $p_T$ < 6 GeV/c.
- K. Ürmössy, T.S. Bíró: PLB689:14 (2010)



# AuAu Collisions at RHIC energies

- Boltzmann Statistics

$$F_h(E) = A\,e^{-\beta E}$$

$$p^0 \frac{dN}{d^3p} \sim \xi m_T K_1(\beta\gamma m_T) I_0(\beta\gamma v p_T) + (1-\xi) p_T K_0(\beta\gamma m_T) I_1(\beta\gamma v p_T) \sim e^{-\beta\gamma(m_T - v p_T)}$$

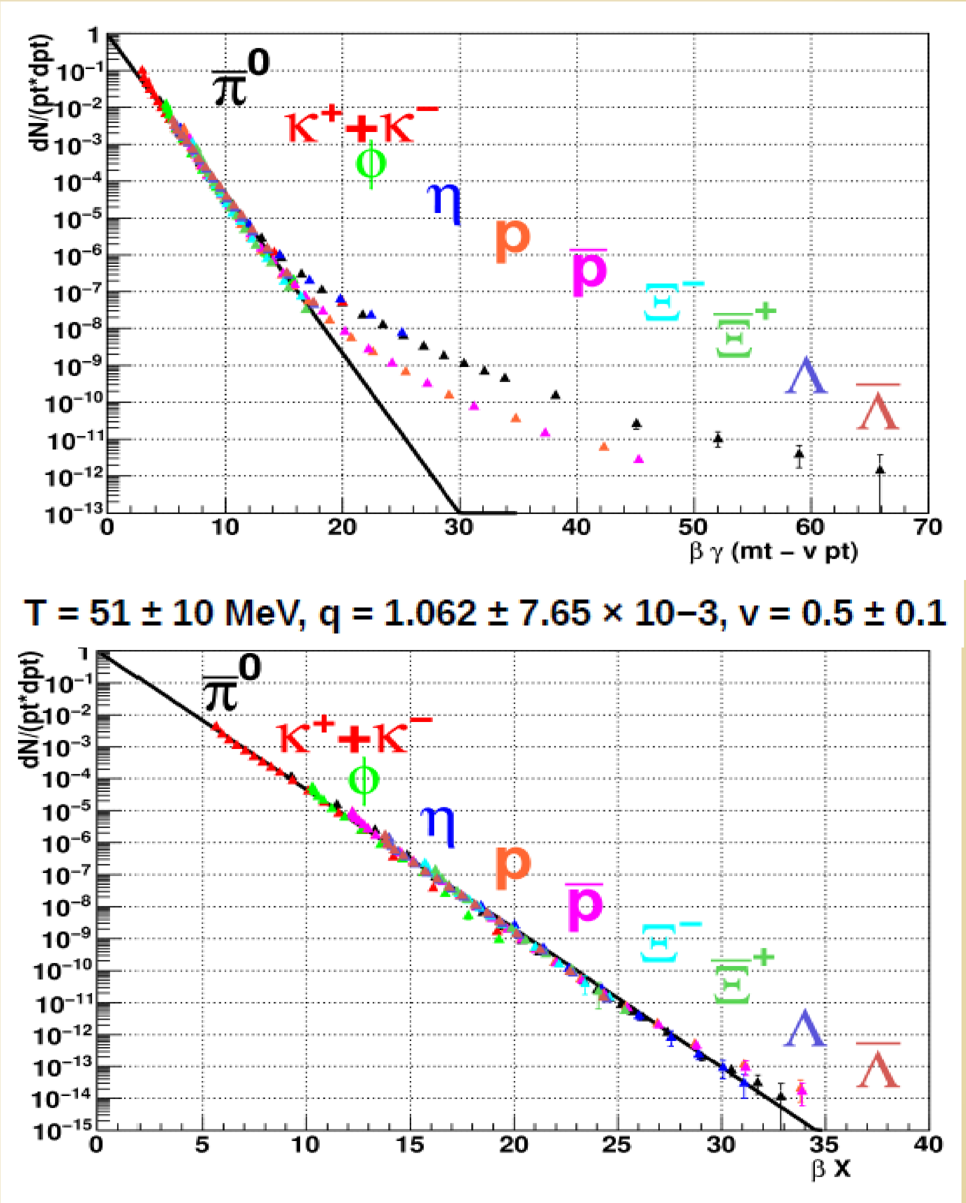
$$\sim e^{-\beta\gamma(m_T - v p_T)}$$

- Non-Extensive (Tsallis)

$$F_h(E) = A\left(1+(q-1)E/T\right)^{-1/(q-1)}$$

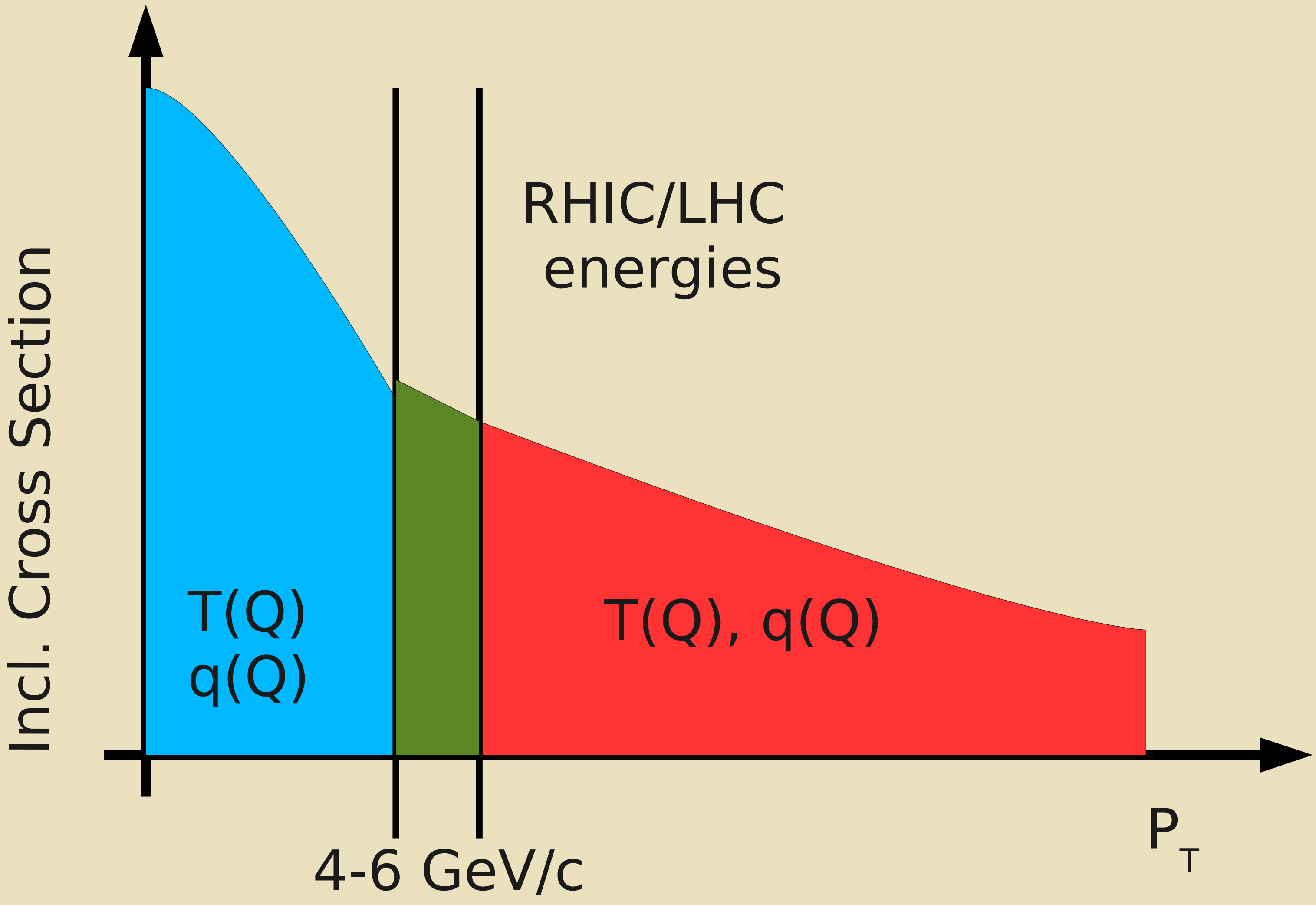
$$p^0 \frac{dN}{d^3p} \sim \frac{\xi m_T G_0(p_T) + (1-\xi) p_T G_2(p_T)}{\left(1+(q-1)\beta\gamma(m_T - v p_T)\right)^{1/(q-1)}}$$

$$\sim p_T^{-q/(q-1)}$$



# $2^{nd}$ way: to resolve the gap...

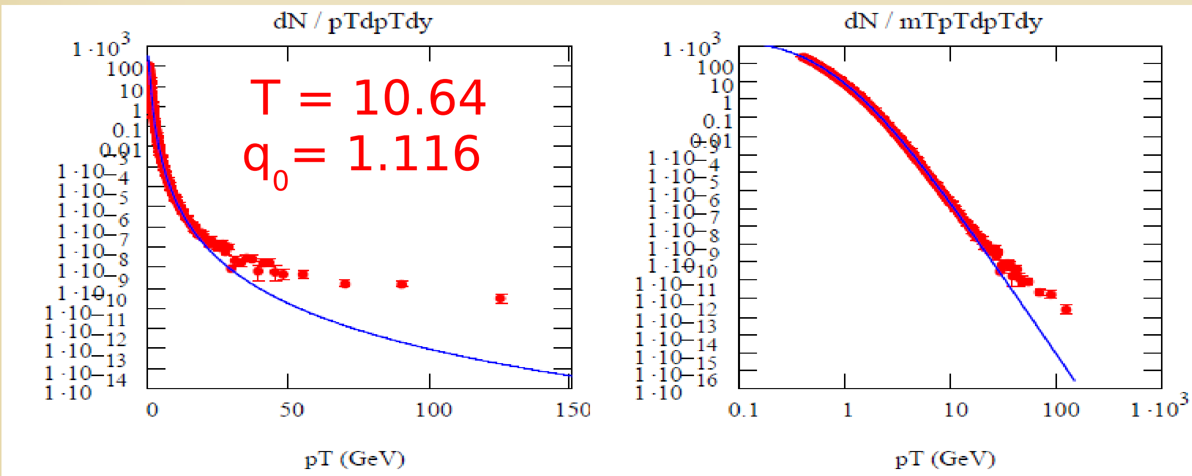
- Suggested interpetation: Tsallis + evolution

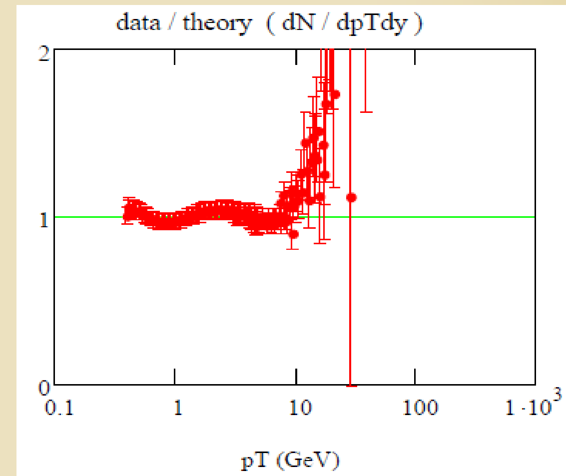


# $2^{nd}$ joint model: Tsallis with evolution

- TEST on CDF ch. hadron data in pp @ 1.96 TeV |y|<1

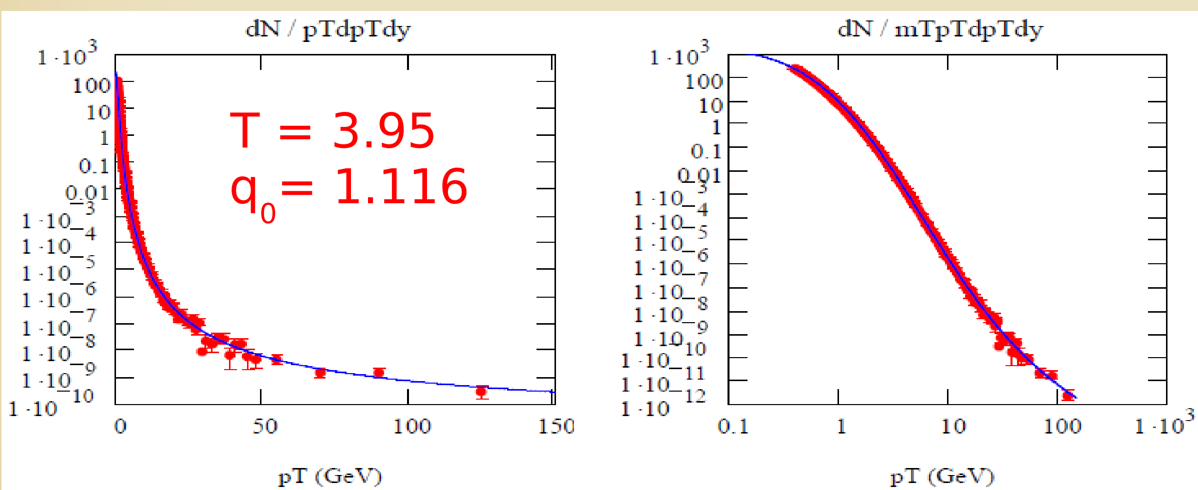
NO evolution

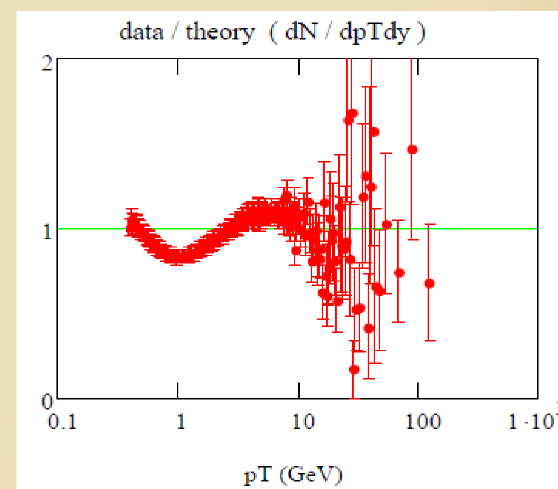



- DGLAP motivated evolution: $n = (q_0-1)^{-1} - 2*\log(\log(Q))$

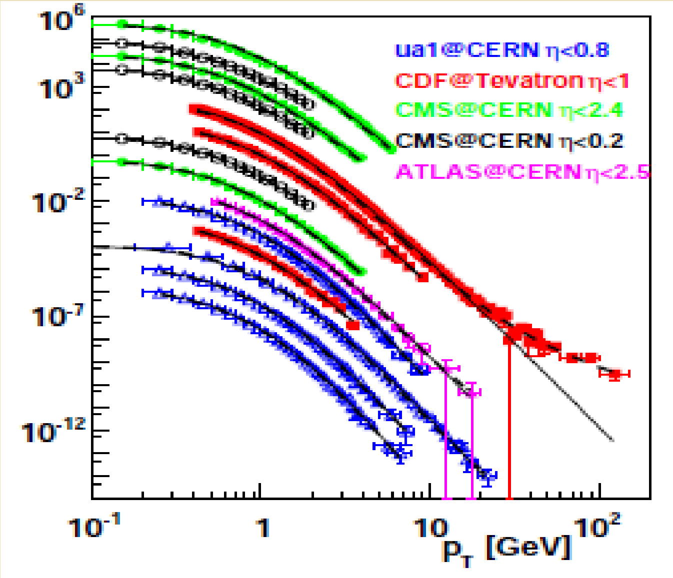
With evolution

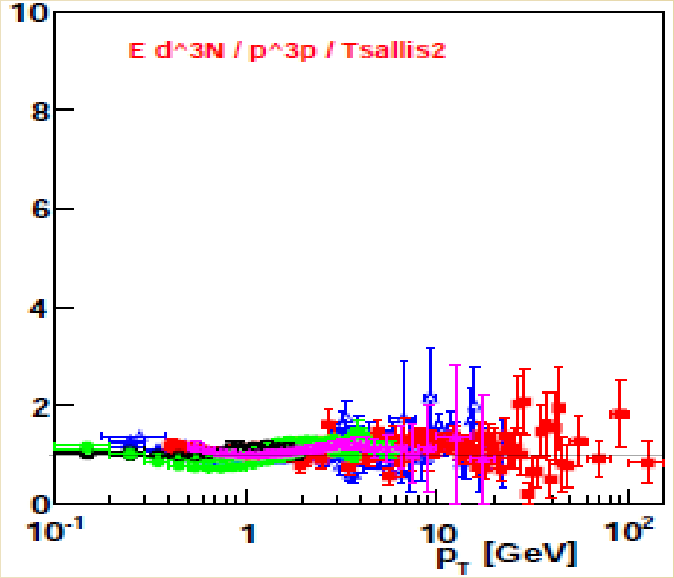


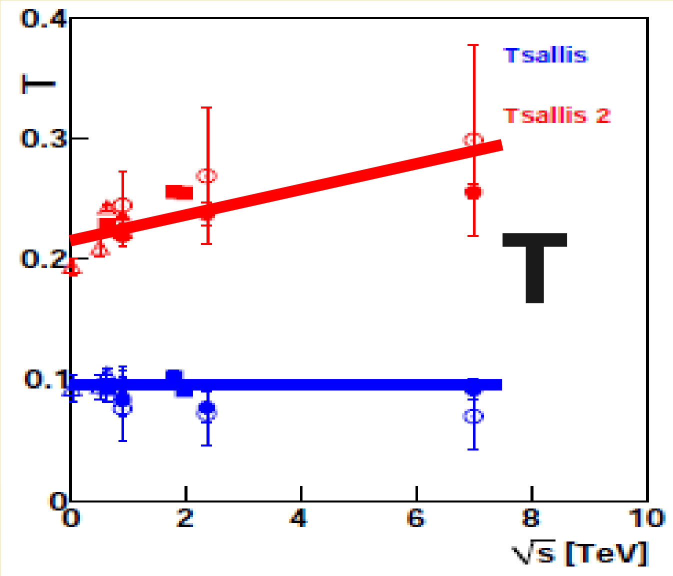
# 2nd joint model: Tsallis with evolution

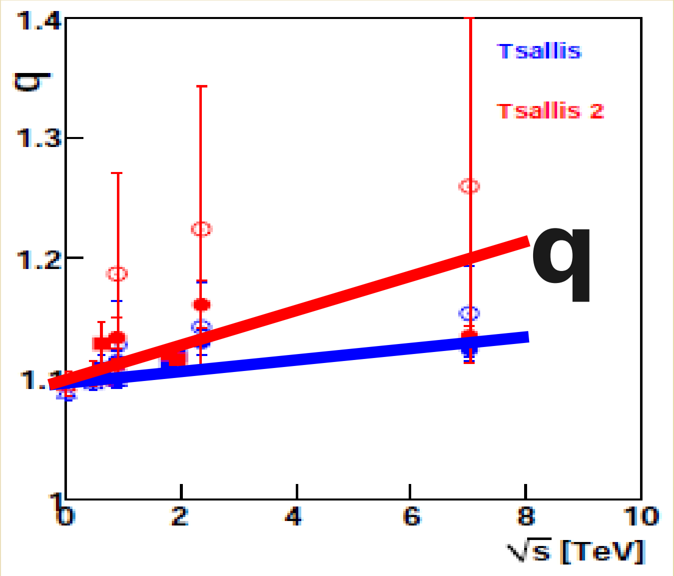
- More TEST: 0.2 - 7 TeV midrapidity data



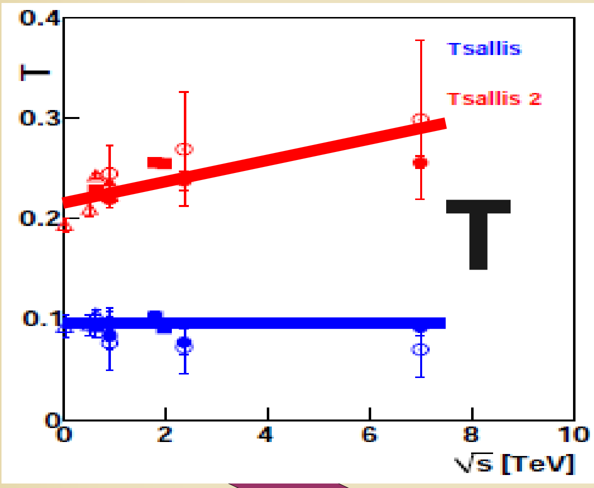

- C.m. Energy dependence of the T & q parameters

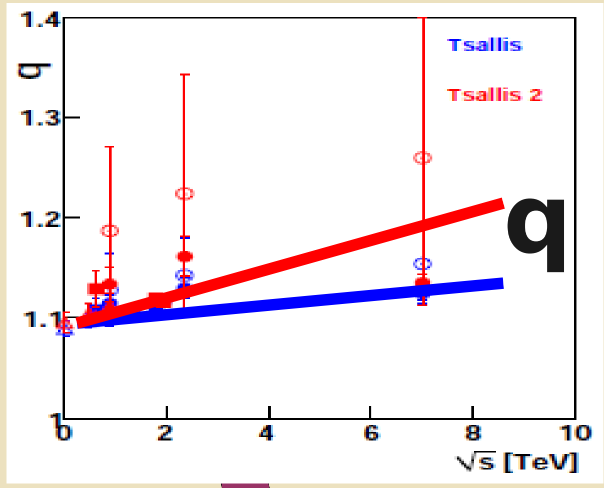


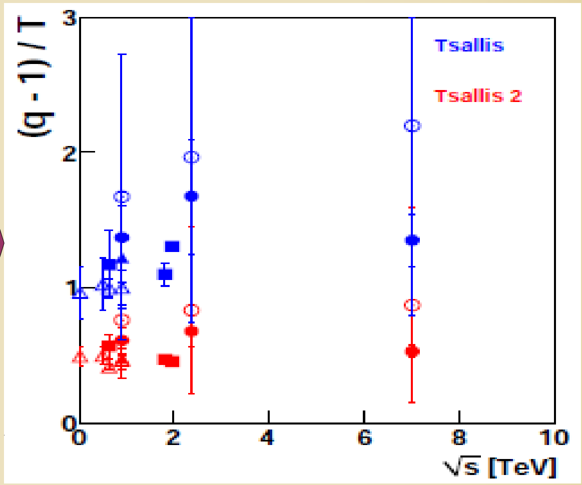



# $2^{nd}$ joint model: Tsallis on q-T plane

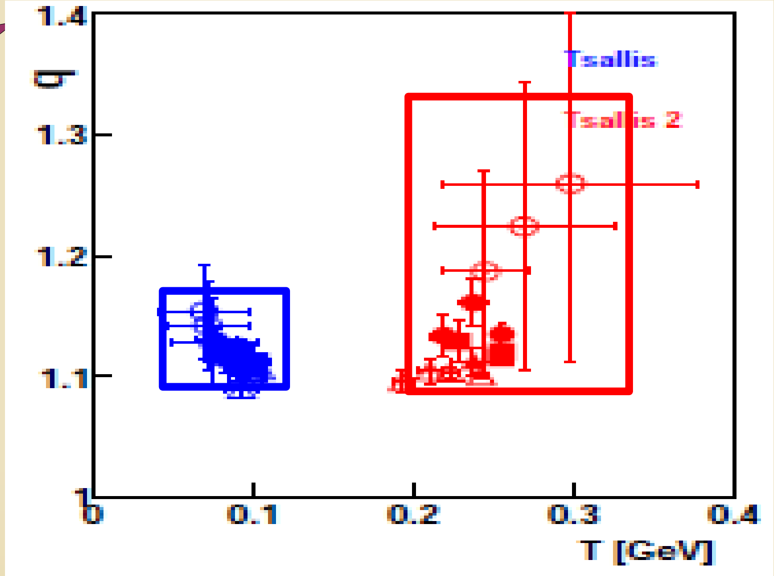
- TEST on various midrapidity data @ different cm



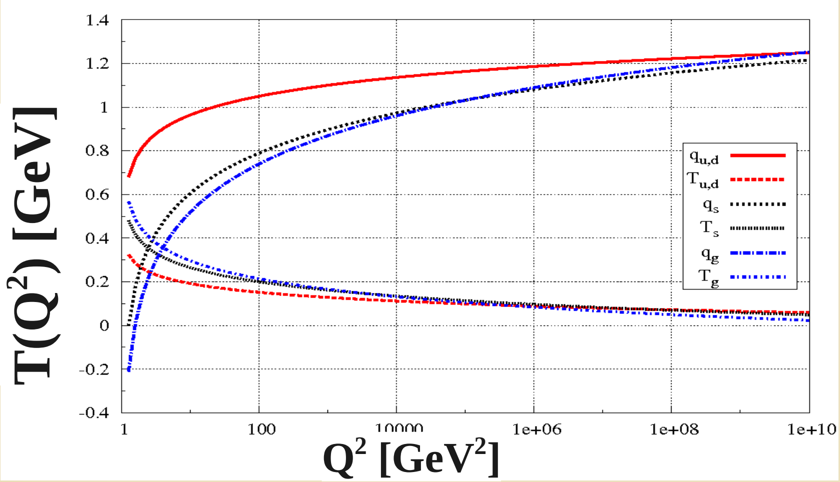



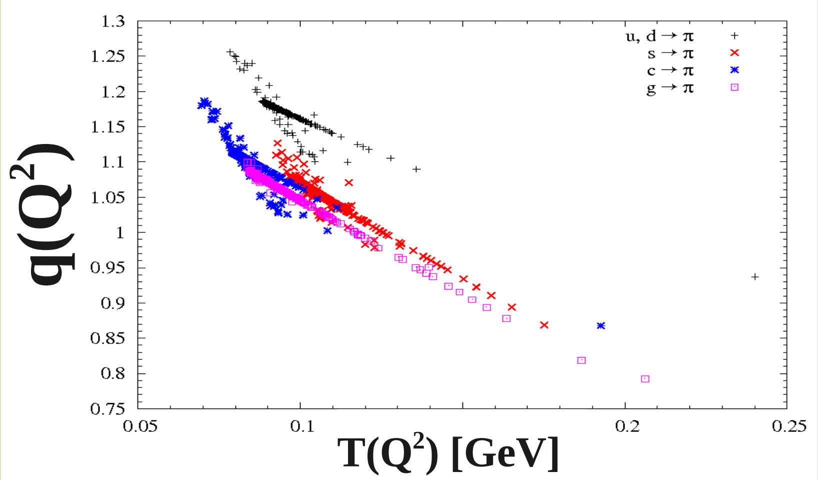

- The q-T plane



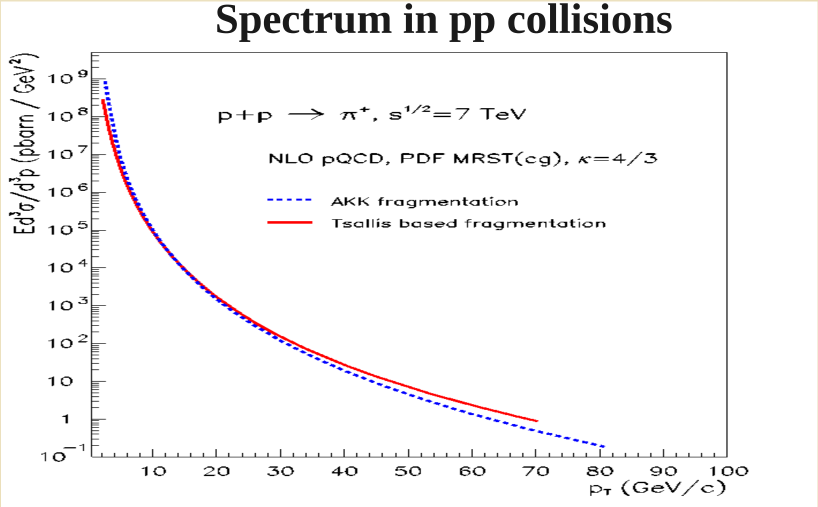
# Test of the Tsallis based FF

- Tsallis T & q parameters in different channels




- Test of the model: pQCD vs. Tsallis based FF
  - pQCD based parton model
    PRC65 034903 (2002)
  - Scalig Tsallis parameters
  - DGLAP evolution
  - Sum Rule for normalization
    GGB et al: Gribov '80 (2010)



# S U M M A R Y

- High & low $p_T$ spectra has different distribution..

  ...however hadronization should not work differently.

- Non-extensive (non-equilibrium) thermodynamic

  Can be applied generally. Based on composition rules, evolution eq. can be obtain even for non-thermalised case.

- Tests and models with Tsallis assumptions

  Seems working for hadron production up to intermediate $p_T$, and extension to the highest $p_T$ region is still question.

  .

# ... so we hope...