Precision Spectroscopy and Collisions of Highly Charged Ions: Fundamental Theories on Trial

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SPARC collaboration

Fundamental interactions

Quantum Field Theory describes:

- Strong
- Electromagnetic
- Weak

Special properties of QED:

- Mediating particle mass is zero: $m_{\rm ph} = 0$
- Coupling constant α is dimensionless and $\alpha \ll 1$

Open questions and possible restrictions of QED:

- Renormalization for bound states
- Perturbation expansion is, most probably, divergent!
  When will we face this problem?
- QED can be an "effective field theory"
  $\Rightarrow$ Physics beyond the Standart Model

Quantum Electrodynamics of free electron

$\alpha = 1/137.036... \ll 1 \Rightarrow$ Perturbation Theory

Magnetic moment of free electron

$$g_{\rm free} = 2\left(1 + \frac{\alpha}{\pi}A^{(2)} + \left(\frac{\alpha}{\pi}\right)^2 A^{(4)} + \left(\frac{\alpha}{\pi}\right)^3 A^{(6)} + \left(\frac{\alpha}{\pi}\right)^4 A^{(8)} + \ldots\right)$$

	2.000 000 000 000
α/π	0.002 322 819 466
$(\alpha/\pi)^2$	−0.000 003 544 610
$(\alpha/\pi)^3$	0.000 000 029 608
$(\alpha/\pi)^4$	−0.000 000 000 101
μ,h,w	0.000 000 000 002
Total	2.002 319 304 361

Test of QED / Determination of α

High-Z few-electron ions

$\alpha = 1/137.036...$, Z – nuclear charge, $V_{\rm nuc}(r) = -\dfrac{\alpha Z}{r}$

Low-Z systems: $\alpha Z \ll 1 \longrightarrow$ expansion αZ

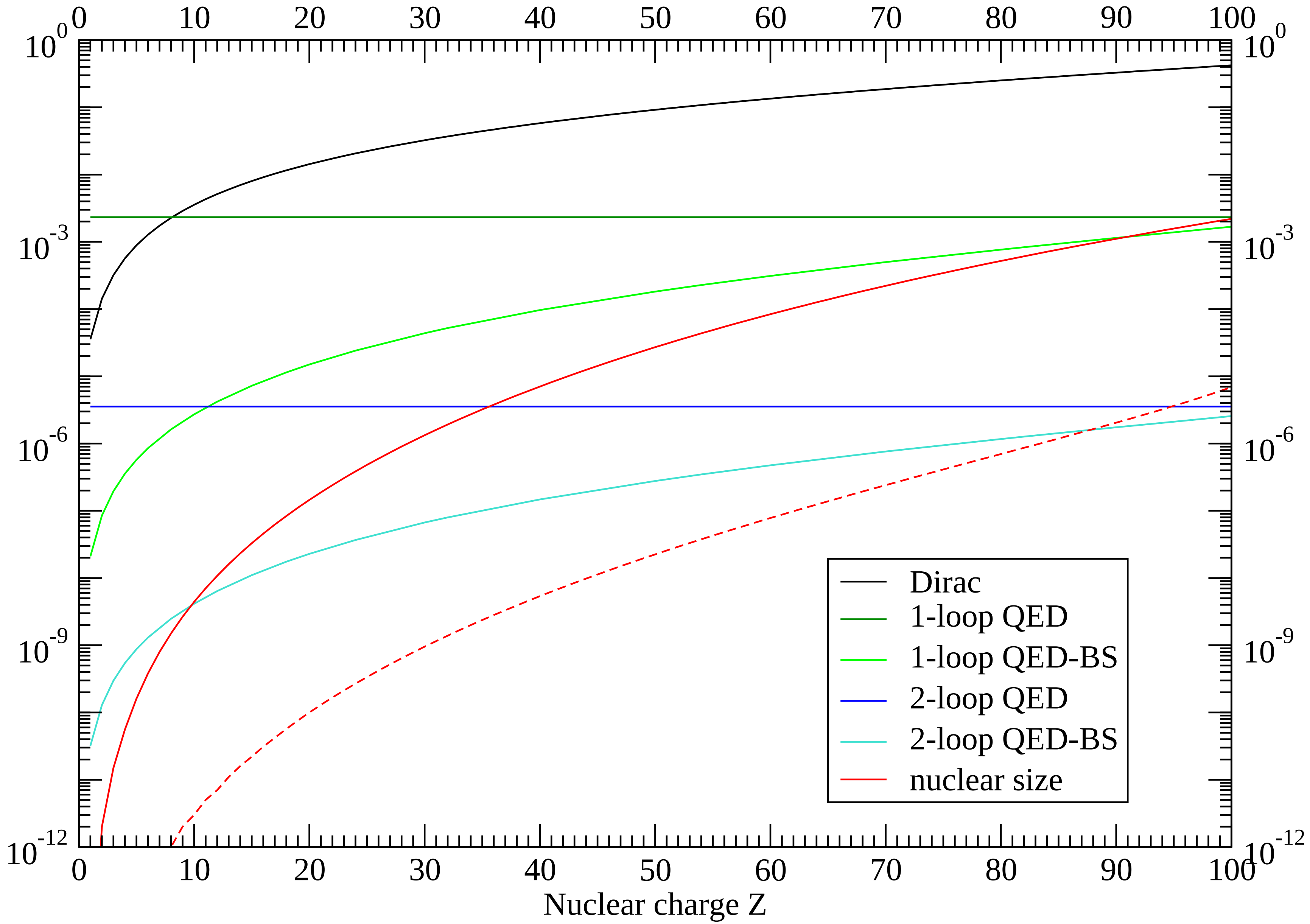
$$\mathcal{E}[\mathrm{H(1s)}] = 10^{10}\mathrm{V/cm}$$
$$\mathcal{E}[\mathrm{Laser}] = 10^{12}\mathrm{V/cm}$$
$$\mathcal{E}[\mathrm{U(1s)}] = 10^{16}\mathrm{V/cm}$$

High-Z systems: $\alpha Z \sim 1 \longrightarrow$ no expansion in αZ — strong-field regime of QED

Magnetic moment of $1s$ electron in H-like ion

$$g_{1s} = 2\left(A^{(0)}(\alpha Z) + \frac{\alpha}{\pi}A^{(2)}(\alpha Z) + \left(\frac{\alpha}{\pi}\right)^2 A^{(4)}(\alpha Z) + \ldots\right) + \Delta g_{\rm nuc}$$

1s g factor



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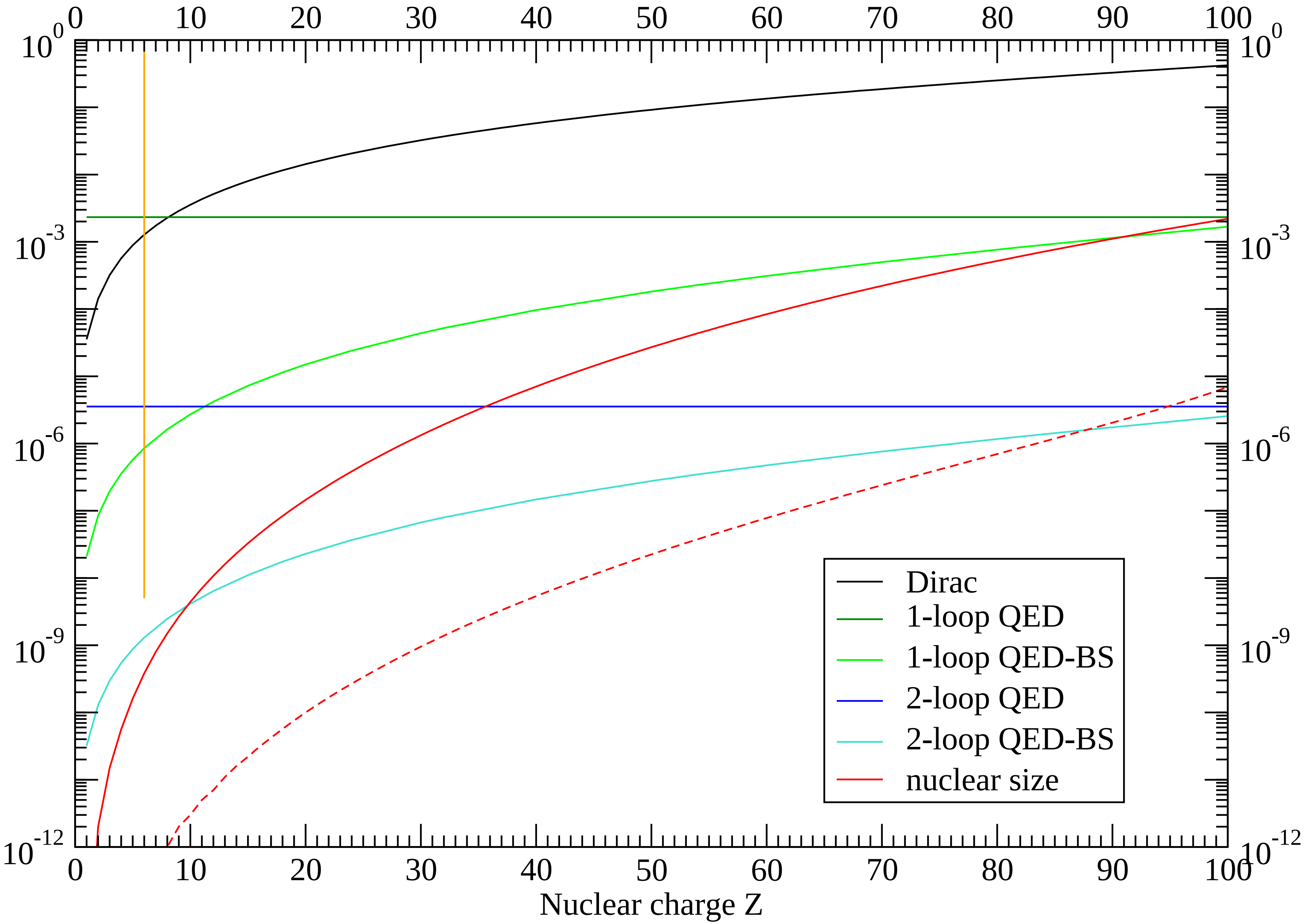
Dirac
1-loop QED
1-loop QED-BS
2-loop QED
2-loop QED-BS
nuclear size
Nuclear charge Z

1s g factor



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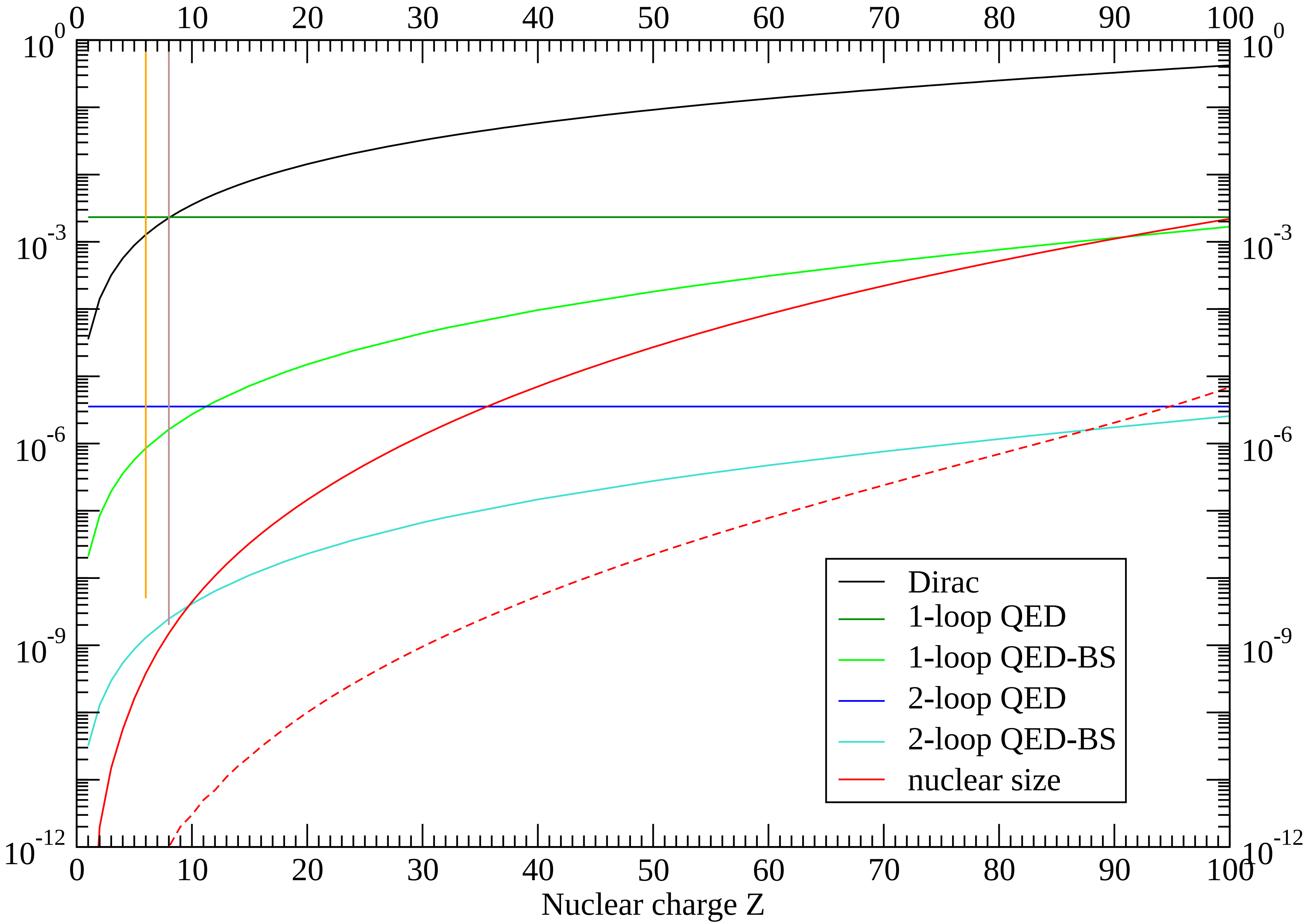

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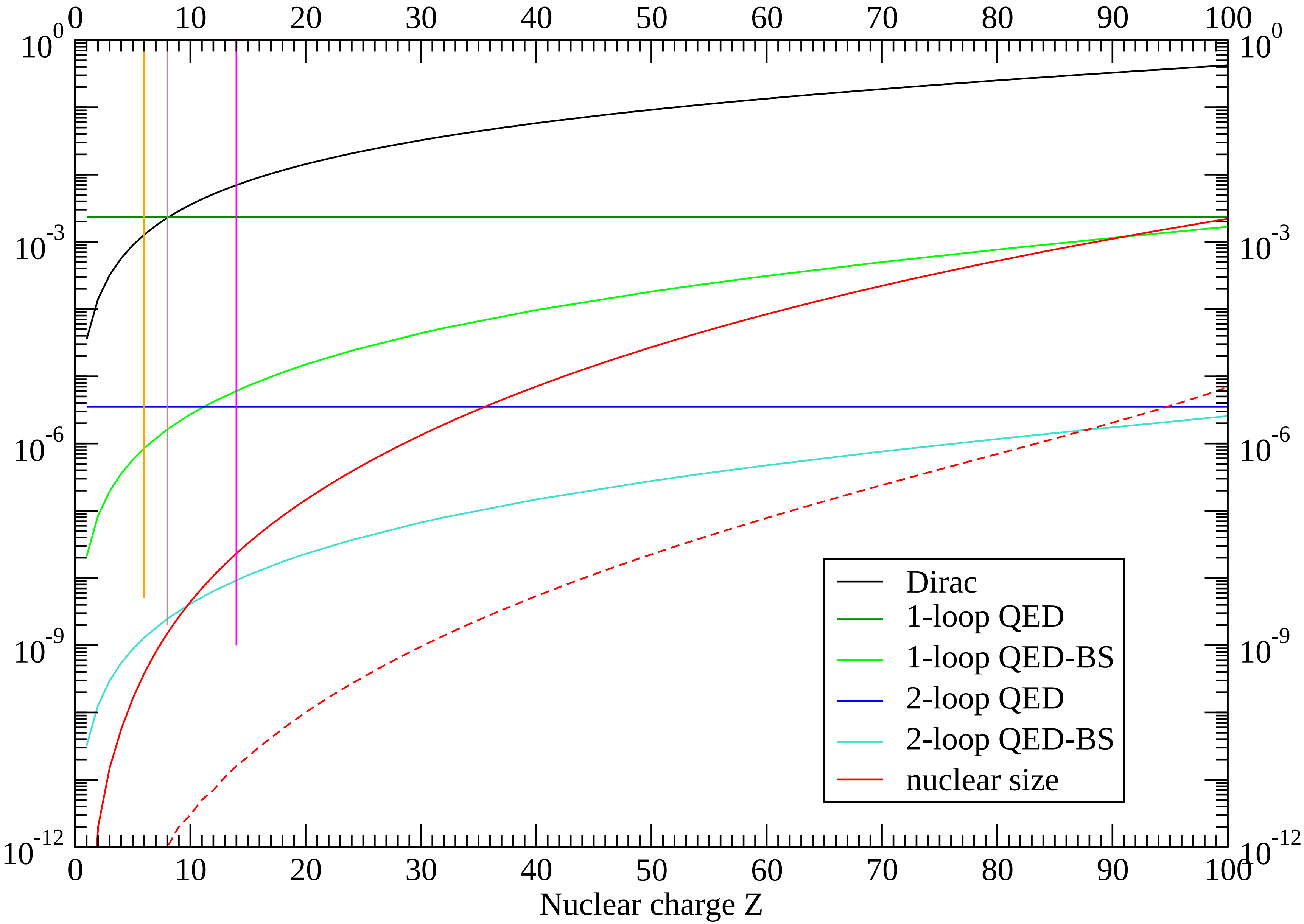

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Nuclear charge Z

Bound-electron g factor

Mainz-GSI collaboration:

2000: $g[^{12}\mathrm{C}^{5+}] =$ 2.001 041 596 (5)

2004: $g[^{16}\mathrm{O}^{7+}] =$ 2.000 047 025 4 (15)

2011: $g[^{28}\mathrm{Si}^{13+}] =$ 1.995 348 951 5 (12)

	1.993 023 571 6 (1)
α/π	0.002 328 682 6 (3)
$(\alpha/\pi)^{2+}$	$-$0.000 003 522 5 (17)
Δg_{rec}	0.000 000 198 8
Δg_{NS}	0.000 000 020 4 (1)
Total	1.995 348 950 8 (18)

Prospects of HITRAP

HITRAP at GSI

- Low-Z ions
  - ▷ Determination of the electron mass m_e/a.u.
- High-Z ions
  - ▷ Determination of α (alternative to g_{free})
- Ions with non-zero nuclear spin
  - ▷ Determination of nuclear magnetic moment

HITRAP at FAIR

- Various isotopes available
- Antiproton experiments

Moreover

- Hyperfine splitting
- Forbidden transitions

All of the above provides verification of bound-state QED with unprecedented accuracy!

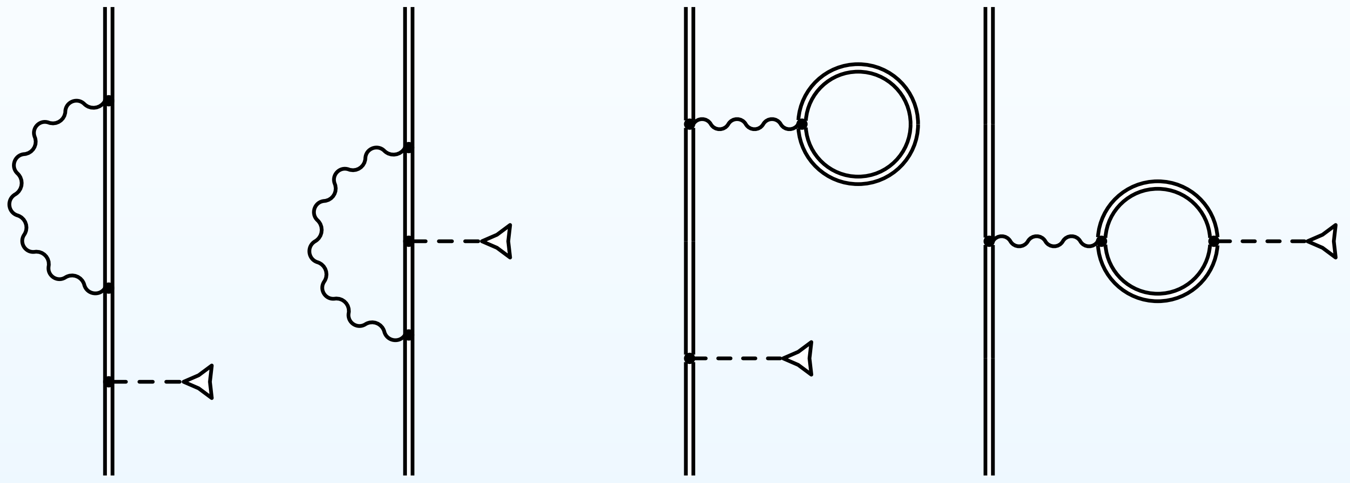
Theoretical status and recent developements

To achieve the required theoretical accuracy for the g factor necessitates a number of elaborate evaluations:

- QED
  - ▷ [α] one-loop QED
    one-electron part
    many-electron part
  - ▷ [α^2] two-loop QED
  - ▷ [α^3] three-loop QED
- Interelectronic interaction
  - ▷ [$1/Z$] one-photon exchange
  - ▷ [$1/Z^2$] two-photon exchange
  - ▷ higher orders: large-scale CI-DFS
- recoil effect
  - $+$ effective potential
  - $+$ QED corrections

One-electron one-loop QED

$\frac{\alpha}{\pi}$:



$$\Delta g_{\rm QED} = \Delta g_{\rm SE} + \Delta g_{\rm VP}$$

Coulomb field:

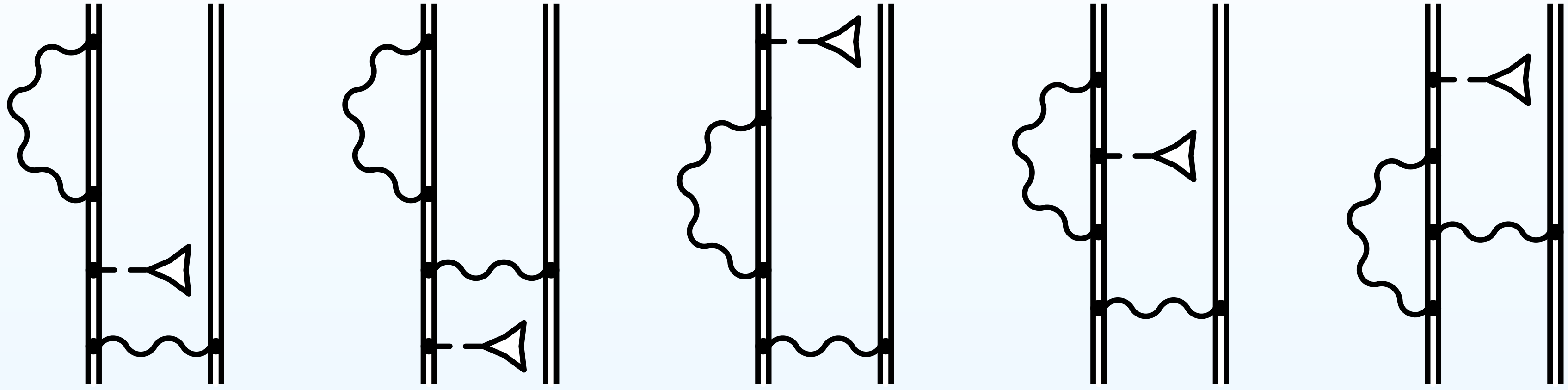
[V. A. Yerokhin et al., PRA (2004)], *[R. N. Lee et al., PRA (2005)]*

Effective screening potential:

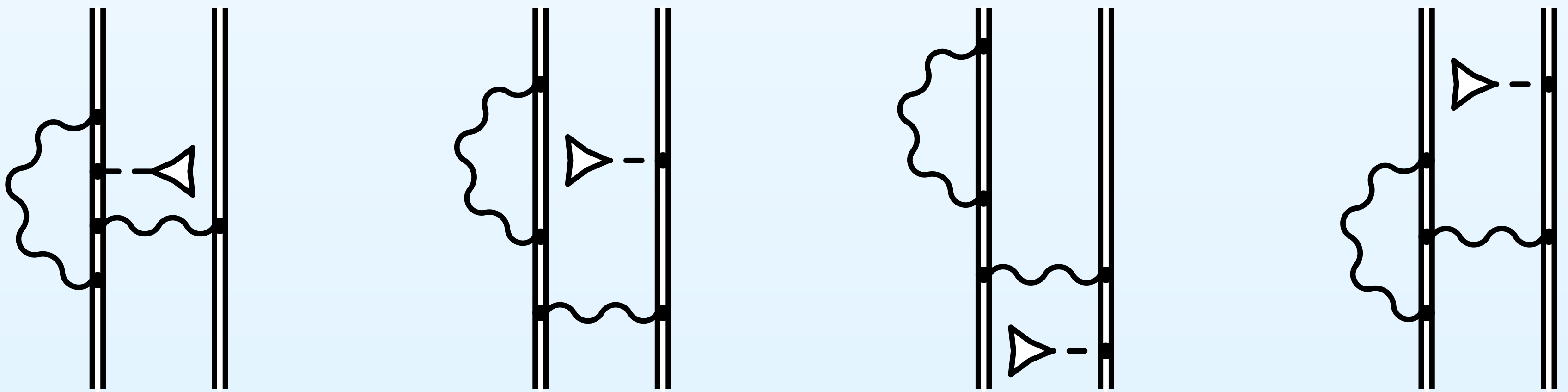
[D. A. Glazov et al., PLA (2006)]

Many-electron one-loop QED

Screened self-energy



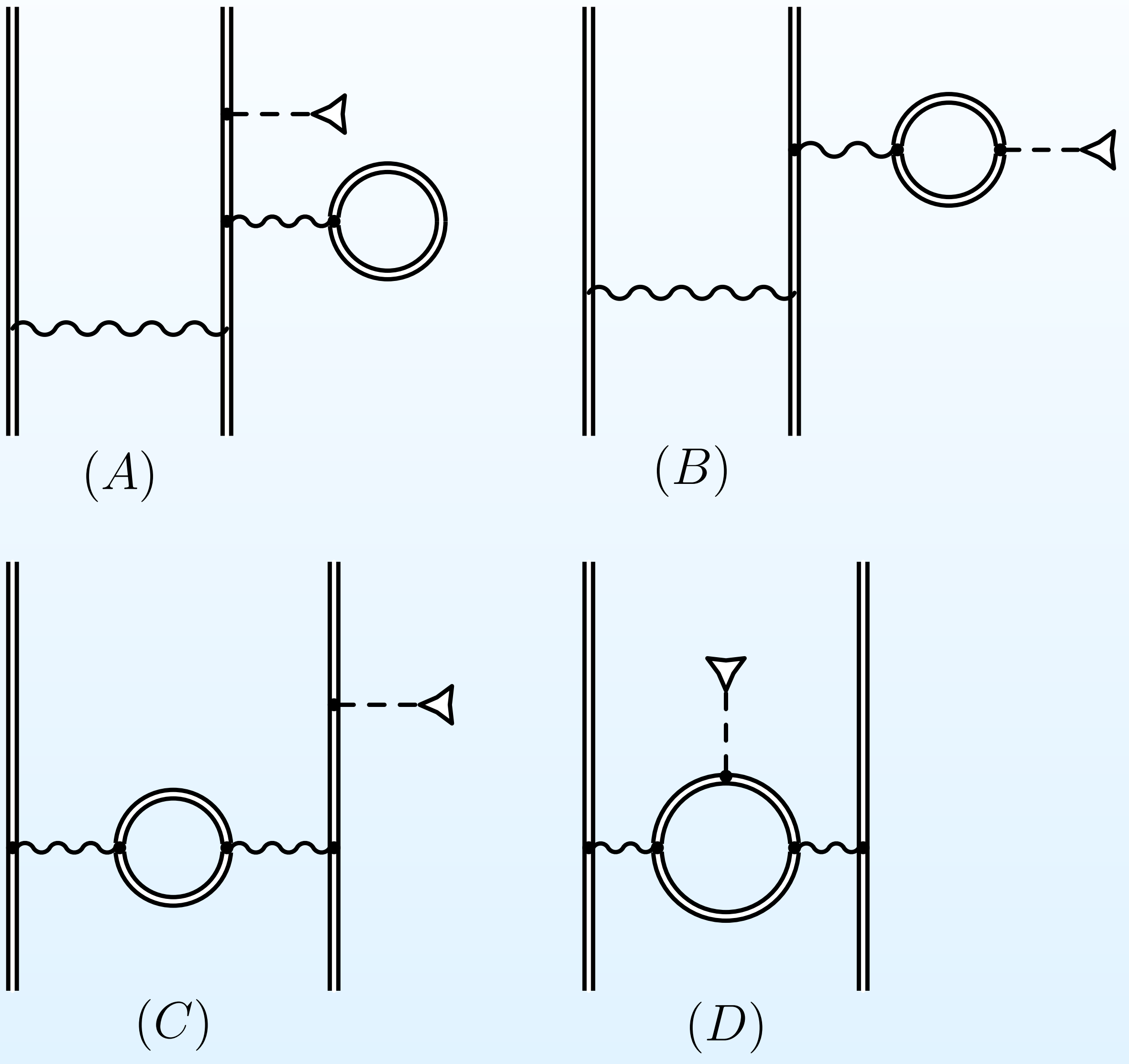
$(A1)$ $(A2)$ (B) $(C1)$ $(C2)$



(D) $(E1)$ $(E2)$ (F)

Many-electron one-loop QED

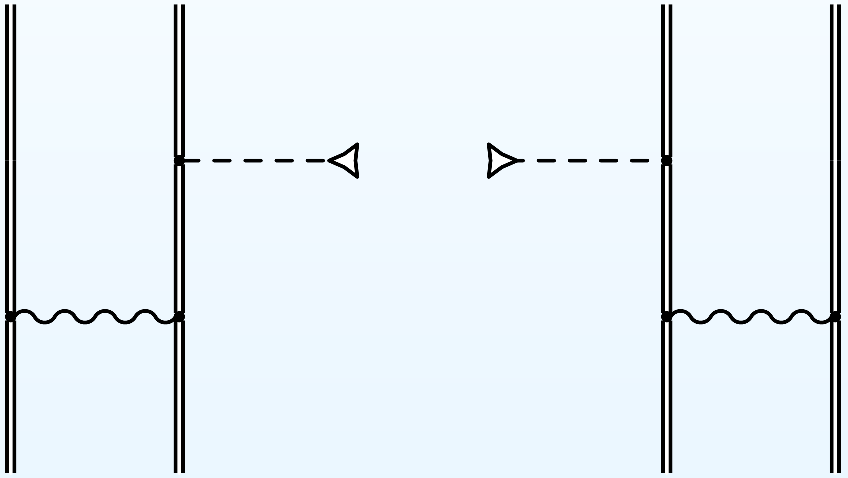
Screened vacuum-polarization



Interelectronic interaction

$$\Delta g_{\rm int} = \frac{1}{Z} B(\alpha Z) + \frac{1}{Z^2} C(\alpha Z) + \ldots$$

$\boldsymbol{\frac{1}{Z}}$ **:**



$1/Z^2$ and higher:

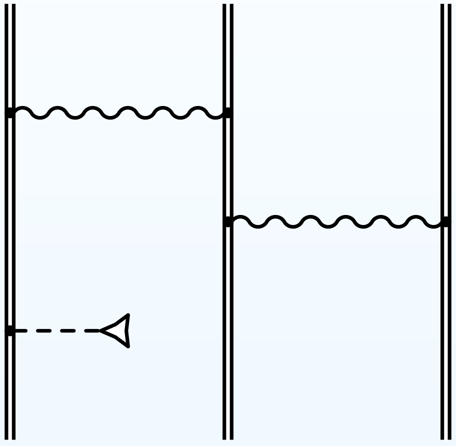
large-scale configuration-interaction Dirac-Fock-Sturm method

Basis: $12s\ 11p\ 10d\ 6f\ 4g\ 2h\ 1i$

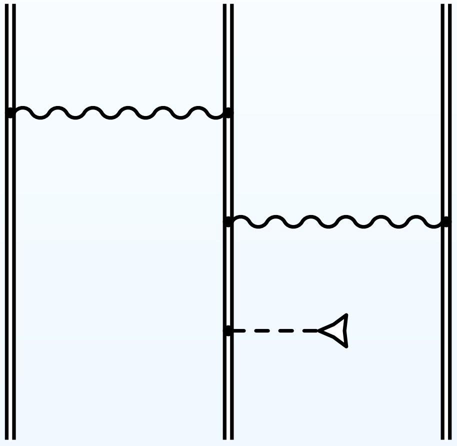
[V. M. Shabaev et al., PRA (2002)], [D. A. Glazov et al., PRA (2004)]

Two-photon exchange

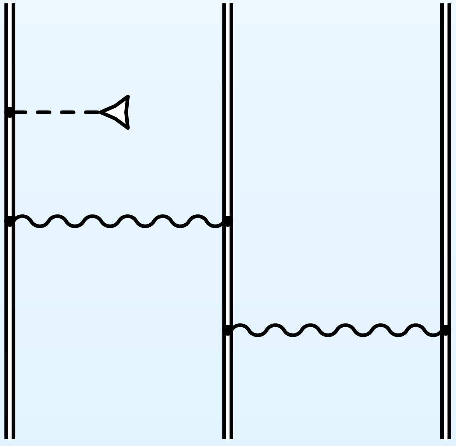
Three-electron diagrams



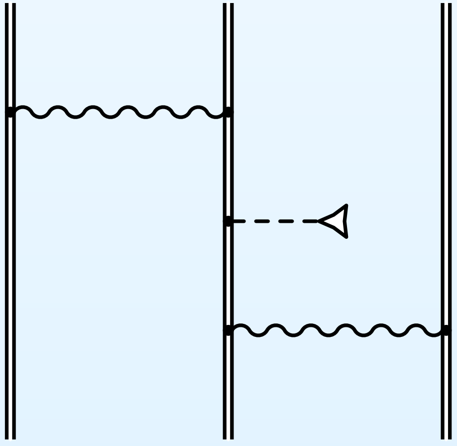
$(1L)$



$(2L)$



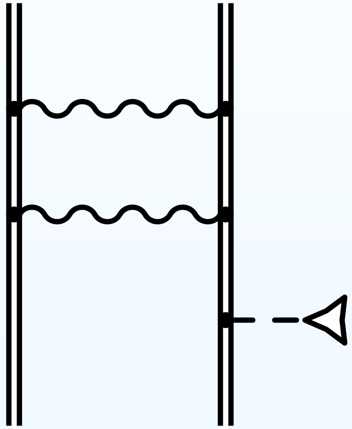
$(1R)$



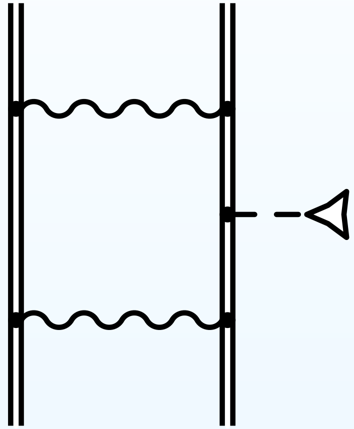
(S)

Two-photon exchange

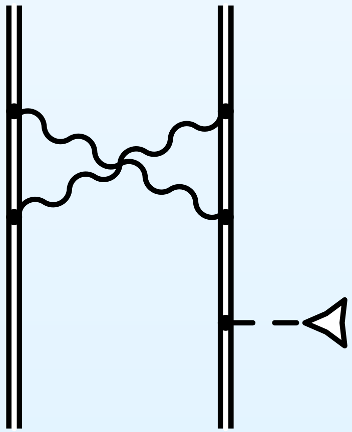
Two-electron diagrams



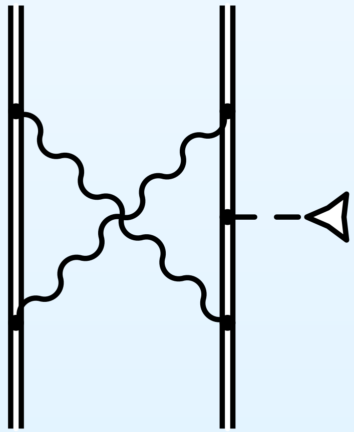
$(\text{ladder}-L)$



$(\text{ladder}-S)$



$(\text{cross}-L)$



$(\text{cross}-S)$

Evaluational procerdure

- Derivation of the general formulae: the two-time Green function method

$$\Delta g^{\mathrm{D}} = \frac{i}{2\pi} \int \mathrm{d}\omega \sum_{n_{1,2,3}} \frac{\langle a n_1 | I(\omega) | n_3 a \rangle \langle n_3 b | I(\Delta) | n_2 b \rangle \langle n_2 | T_0 | n_1 \rangle}{(\varepsilon_a - \omega - \varepsilon_{\overline{n_1}})(\varepsilon_a - \omega - \varepsilon_{\overline{n_2}})(\varepsilon_a - \omega - \varepsilon_{\overline{n_3}})}$$

- Ultraviolet divergencies: potential expansion
  - 0,1-potential terms: momentum space
  - many-potential term: coordinate space
- Infrared divergencies:
  - proof the cancellation analytically
  - evaluate the sum numerically
- Numerical procedure:
  - partial-wave expansion
  - finite basis set: DKB-splines
- Feynman and Coulomb gauges

g factor of Li-like Pb

Dirac value (point nucleus)	1.932 002 904
e^-–e^- interaction, $1/Z$	0.002 148 29
e^-–e^- interaction, $1/Z^{2+}$	0.000 007 6 (27)
QED, one-loop	0.002 411 7 (1)
QED, two-loop	-0.000 003 6 (5)
QED, screening	-0.000 001 8 (1)
Recoil	0.000 000 2 (3)
Nuclear size	0.000 078 6 (1)
Nuclear polarization	-0.000 000 04 (2)
Total theory	1.936 628 7 (28)

g factor of Li-like U

Dirac value (point nucleus)	1.910 722 624
e^-–e^- interaction, $1/Z$	0.002 509 84
e^-–e^- interaction, $1/Z^{2+}$	0.000 008 5 (38)
QED, one-loop	0.002 446 3 (2)
QED, two-loop	-0.000 003 6 (8)
QED, screening	-0.000 001 7 (1)
Recoil	0.000 000 3 (7)
Nuclear size	0.000 241 3 (4)
Nuclear polarization	-0.000 000 2 7 (14)
Total theory	1.915 904 9 (40)

Hyperfine splitting of Li-like Bi

$$\Delta' E = \Delta E[(1s)^2 2s] - \xi \Delta E[1s]$$

	$\Delta E[(1s)^2 2s]$	$\xi \Delta E[1s]$	$\Delta' E$
Dirac value	844.829	876.638	-31.809
Interelectronic interaction	-29.75(4)		-29.75(4)
QED	-5.052	-5.088	0.036
	0.21(4)		0.21(4)
QED, screening	0.194(6)		0.194(6)
			-61.31(6)
Total theory			-61.32(4)

[A. V. Volotka et al., PRL (2009)]

Low-energy collisions of antiprotons with heavy ions

Coulomb glory effect:

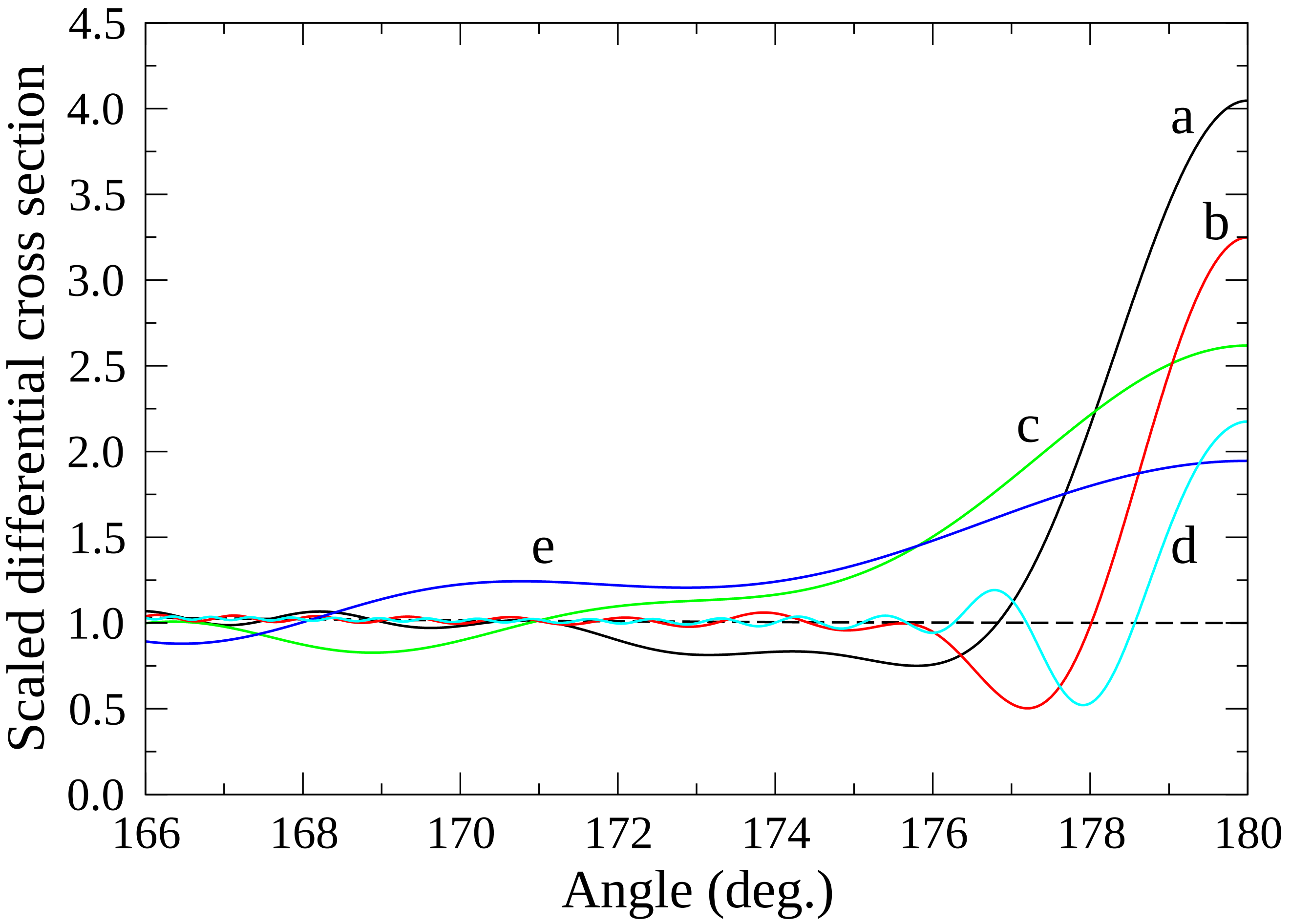
a prominent maximum of the differential cross section in the backward direction at some energies of the incident particle.
The effect is caused by a screening of the Coulomb potential of the nucleus.
[Yu.N. Demkov, V.N. Ostrovsky, D.A. Telnov, JETP (1984)]

- for few-electron ions
  electron screening
- for bare nuclei
  vacuum-polarization screening
  Direct evidence for this QED effect!

Calculations performed for bare, He-, Ne-, Ni-like and neutral uranium

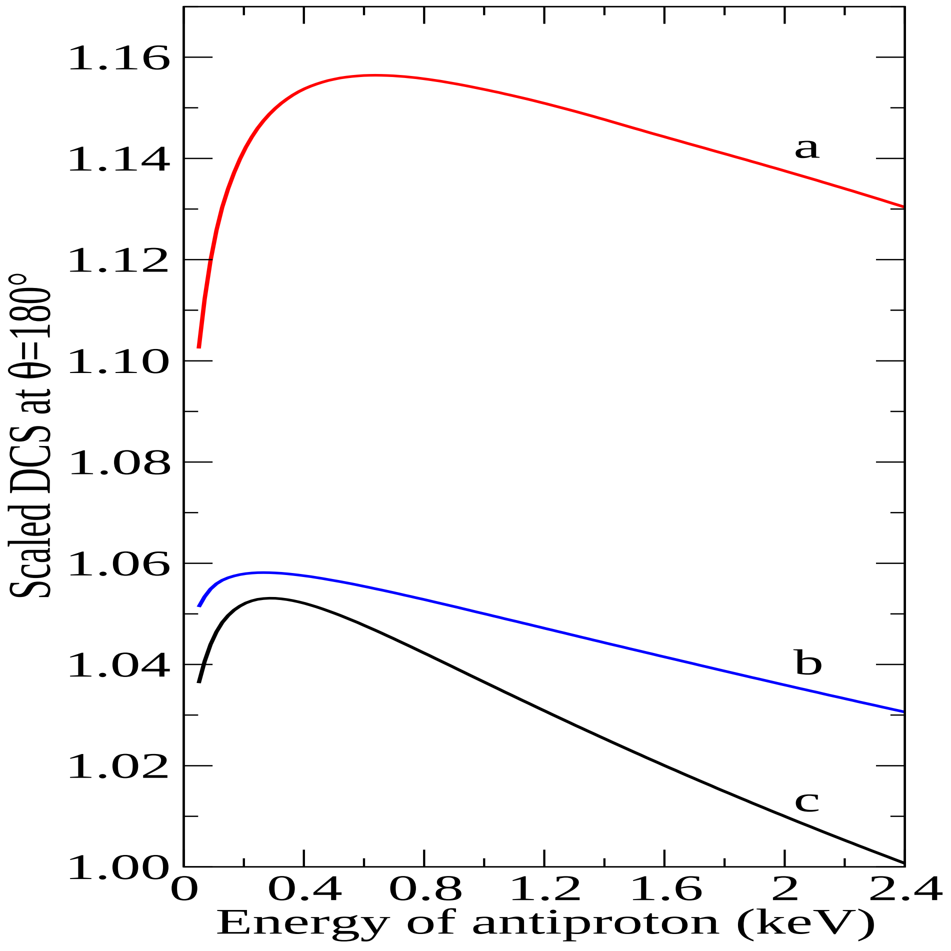
[A.V. Maiorova et al., PRA (2007); A.V. Maiorova et al., JPB (2009)].

Coulomb glory effect for He-like uranium



$E =$5 eV (d), 20 eV (b), 100 eV (a), 500 eV (c), 1000 eV (e)

Coulomb glory effect for bare uranium



non-rel.: fns = 1, VP = (c)

rel.: fns = (b), VP = (a)

thank you for your attention!

questions are welcome!

mind the poster on heavy ions collisions presented by Yury Kozhedub!