Theoretical prerequisites for high-precision atomic experiments at HITRAP

Dmitry A. Glazov

St. Petersburg State University

SPARC collaboration

Prospects of HITRAP

Trapped charged particles nearly at rest

- g factor
- Hyperfine splitting
- Forbidden transitions

HITRAP at GSI

- Low-Z ions
  - ▷ Determination of the electron mass $m_e/$a.u.
- High-Z ions
  - ▷ Determination of α (alternative to g_{free})
- Ions with non-zero nuclear spin
  - ▷ Determination of nuclear magnetic moments

HITRAP at FAIR

- Various isotopes available
- Antiproton experiments
- ...

Quantum Electrodynamics of free electron

$\alpha = 1/137.036... \ll 1 \Rightarrow$ Perturbation Theory

Magnetic moment of free electron

$$g_{\rm free} = 2\left(1 + \frac{\alpha}{\pi}A^{(2)} + \left(\frac{\alpha}{\pi}\right)^2 A^{(4)} + \left(\frac{\alpha}{\pi}\right)^3 A^{(6)} + \left(\frac{\alpha}{\pi}\right)^4 A^{(8)} + \dots\right)$$

	2.000 000 000 000
α/π	0.002 322 819 466
$(\alpha/\pi)^2$	$-$0.000 003 544 610
$(\alpha/\pi)^3$	0.000 000 029 608
$(\alpha/\pi)^4$	$-$0.000 000 000 101
μ,h,w	0.000 000 000 002
Total	2.002 319 304 361

High-Z few-electron ions

$\alpha = 1/137.036...$, Z – nuclear charge, $V_{\rm nuc}(r) = -\dfrac{\alpha Z}{r}$

Low-Z systems: $\alpha Z \ll 1 \longrightarrow$ expansion αZ

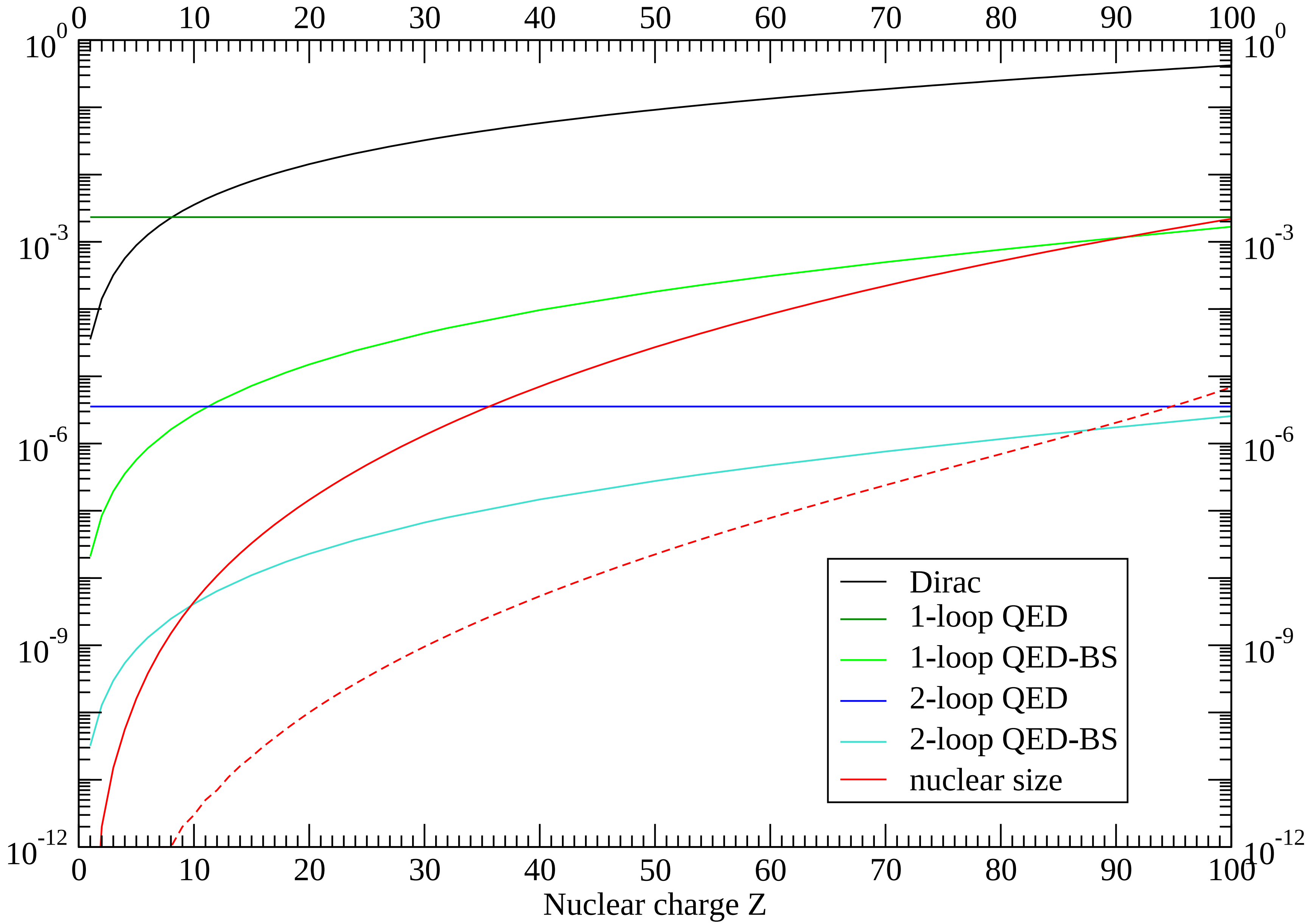
$$\mathcal{E}[\mathrm{H(1s)}] = 10^{10}\mathrm{V/cm}$$
$$\mathcal{E}[\mathrm{Laser}] = 10^{12}\mathrm{V/cm}$$
$$\mathcal{E}[\mathrm{U(1s)}] = 10^{16}\mathrm{V/cm}$$

High-Z systems: $\alpha Z \sim 1 \longrightarrow$ no expansion in αZ — strong-field regime of QED

Magnetic moment of $1s$ electron in H-like ion

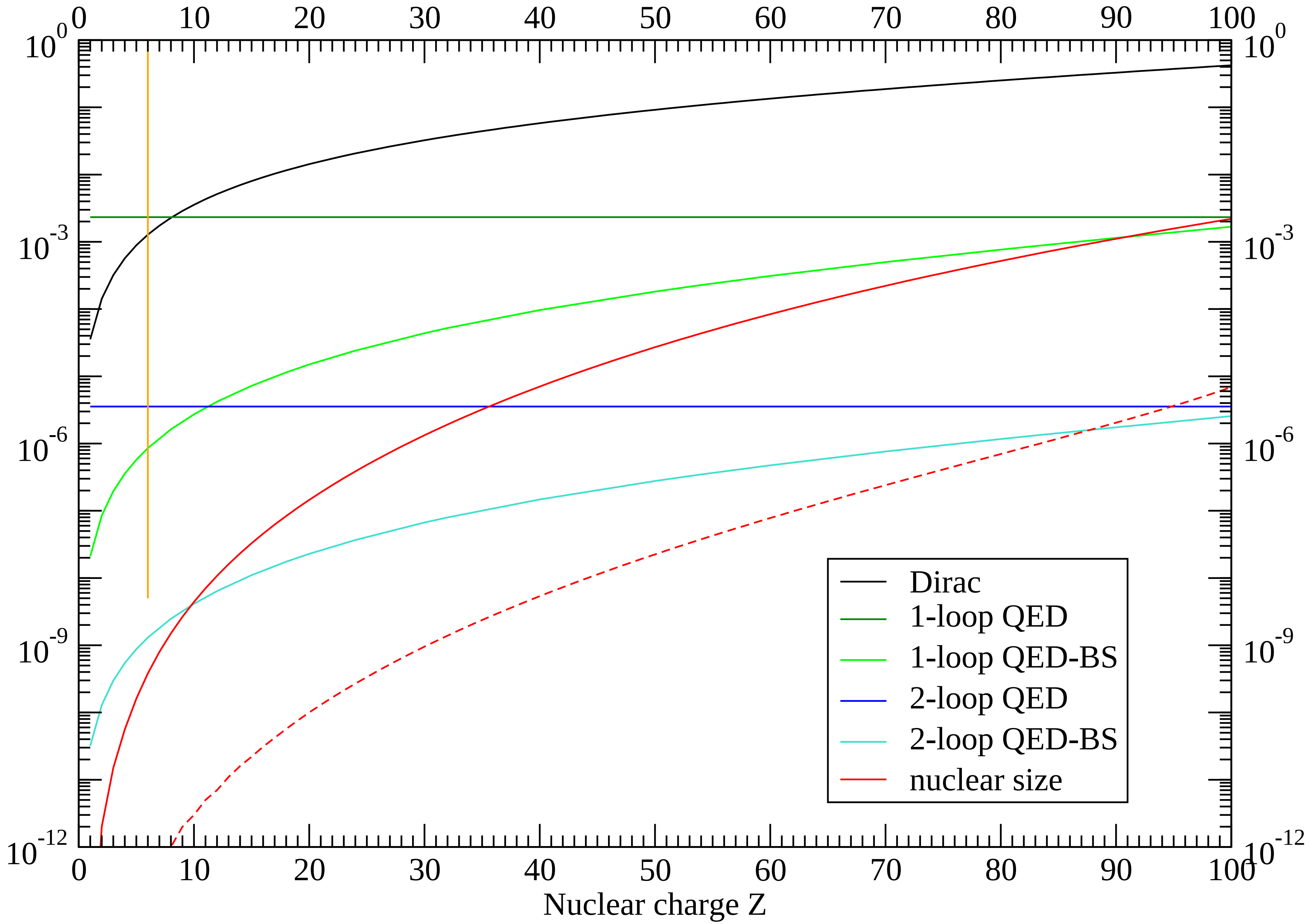
$$g_{1s} = 2\left(A^{(0)}(\alpha Z) + \frac{\alpha}{\pi} A^{(2)}(\alpha Z) + \left(\frac{\alpha}{\pi}\right)^2 A^{(4)}(\alpha Z) + \ldots \right) + \Delta g_{\rm nuc}$$

1s g factor



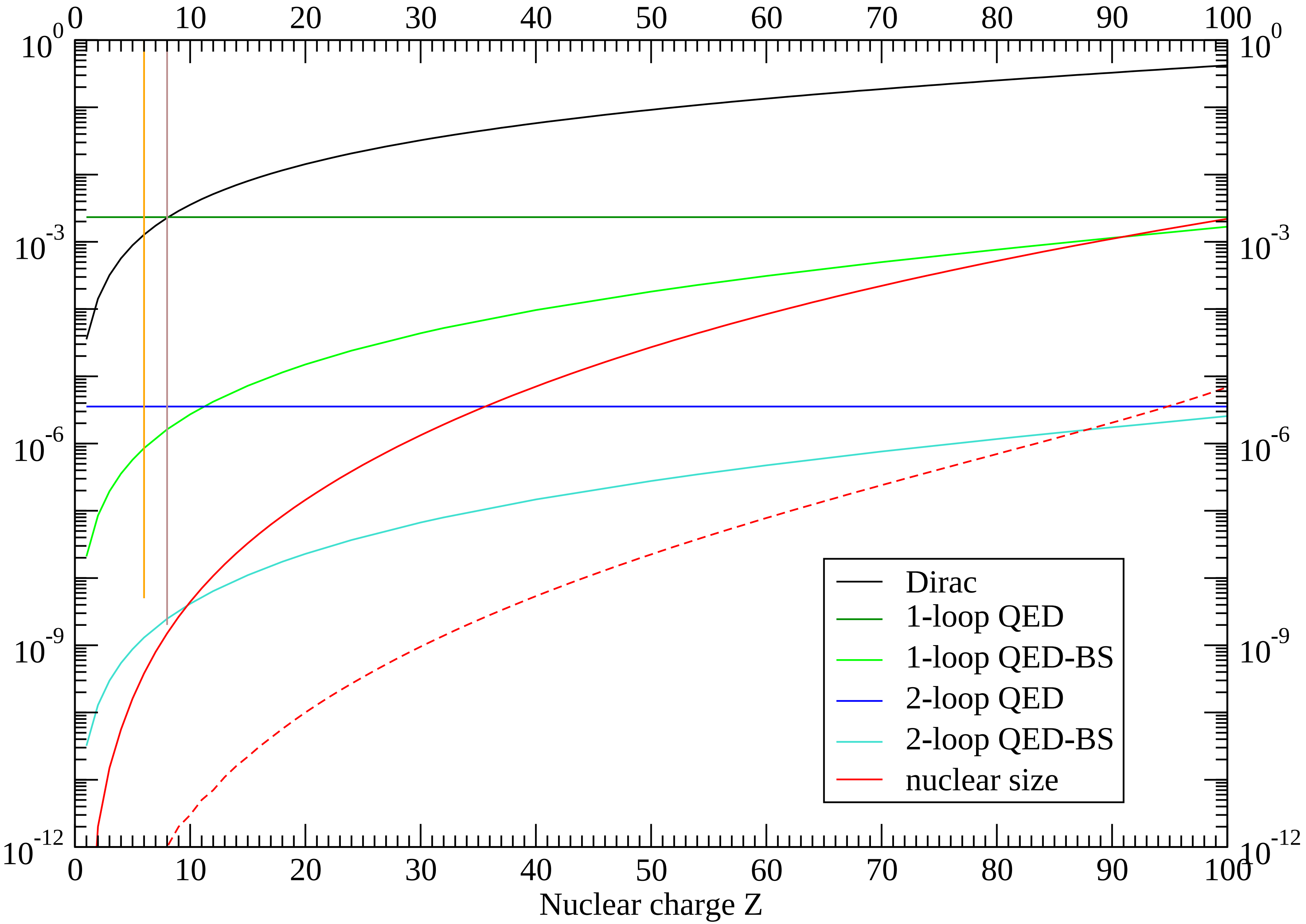
0
10
20
30
40
50
60
70
80
90
100
10^0
10^-3
10^-6
10^-9
10^-12
Dirac
1-loop QED
1-loop QED-BS
2-loop QED
2-loop QED-BS
nuclear size
Nuclear charge Z

1s g factor



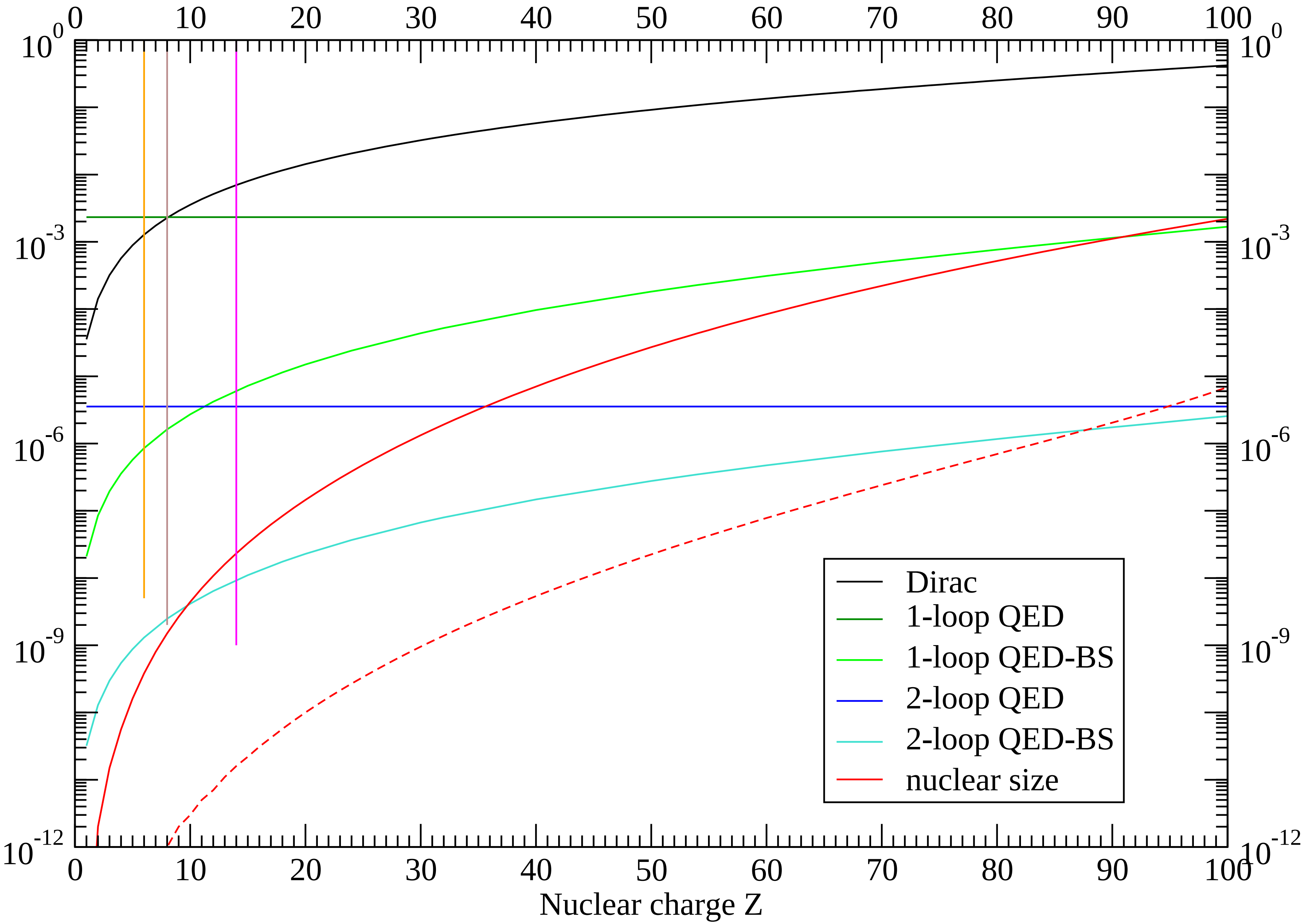
0
10
20
30
40
50
60
70
80
90
100
10^{0}
10^{-3}
10^{-6}
10^{-9}
10^{-12}
Dirac
1-loop QED
1-loop QED-BS
2-loop QED
2-loop QED-BS
nuclear size
Nuclear charge Z

1s g factor



0
10
20
30
40
50
60
70
80
90
100
10^{0}
10^{-3}
10^{-6}
10^{-9}
10^{-12}
Nuclear charge Z
Dirac
1-loop QED
1-loop QED-BS
2-loop QED
2-loop QED-BS
nuclear size

1s g factor



0
10
20
30
40
50
60
70
80
90
100
10^{0}
10^{-3}
10^{-6}
10^{-9}
10^{-12}
Dirac
1-loop QED
1-loop QED-BS
2-loop QED
2-loop QED-BS
nuclear size
Nuclear charge Z

Bound-electron g factor

Mainz-GSI collaboration:

2000: $g[^{12}\mathrm{C}^{5+}] =$ 2.001 041 596 (5)

2004: $g[^{16}\mathrm{O}^{7+}] =$ 2.000 047 025 4 (15)

2011: $g[^{28}\mathrm{Si}^{13+}] =$ 1.995 348 951 5 (12)

	1.993 023 571 6 (1)
α/π	0.002 328 682 6 (3)
$(\alpha/\pi)^{2+}$	$-$0.000 003 522 5 (17)
Δg_{rec}	0.000 000 198 8
Δg_{NS}	0.000 000 020 4 (1)
Total	1.995 348 950 8 (18)

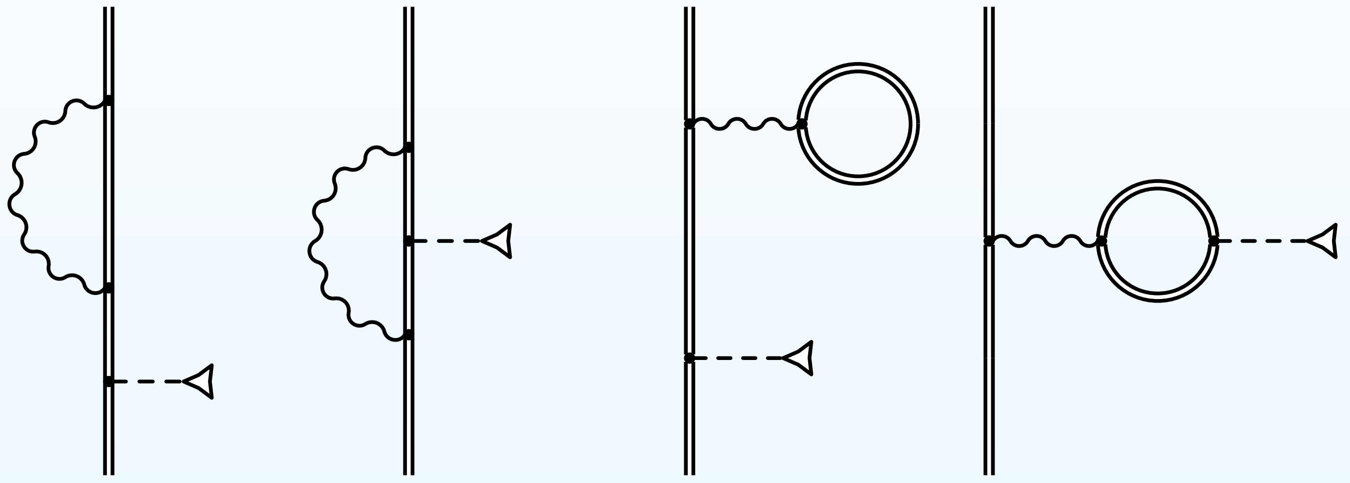
Theoretical status and recent developements

To achieve the required theoretical accuracy for the g factor necessitates a number of elaborate evaluations:

- QED
  - ▷ [α] one-loop QED
    one-electron part
    many-electron part
  - ▷ [α^2] two-loop QED
  - ▷ [α^3] three-loop QED
- Interelectronic interaction
  - ▷ [$1/Z$] one-photon exchange
  - ▷ [$1/Z^2$] two-photon exchange
  - ▷ higher orders: large-scale CI-DFS
- recoil effect
  - $+$ effective potential
  - $+$ QED corrections

One-electron one-loop QED

$\frac{\alpha}{\pi}$:



$$\Delta g_{\rm QED} = \Delta g_{\rm SE} + \Delta g_{\rm VP}$$

Coulomb field:

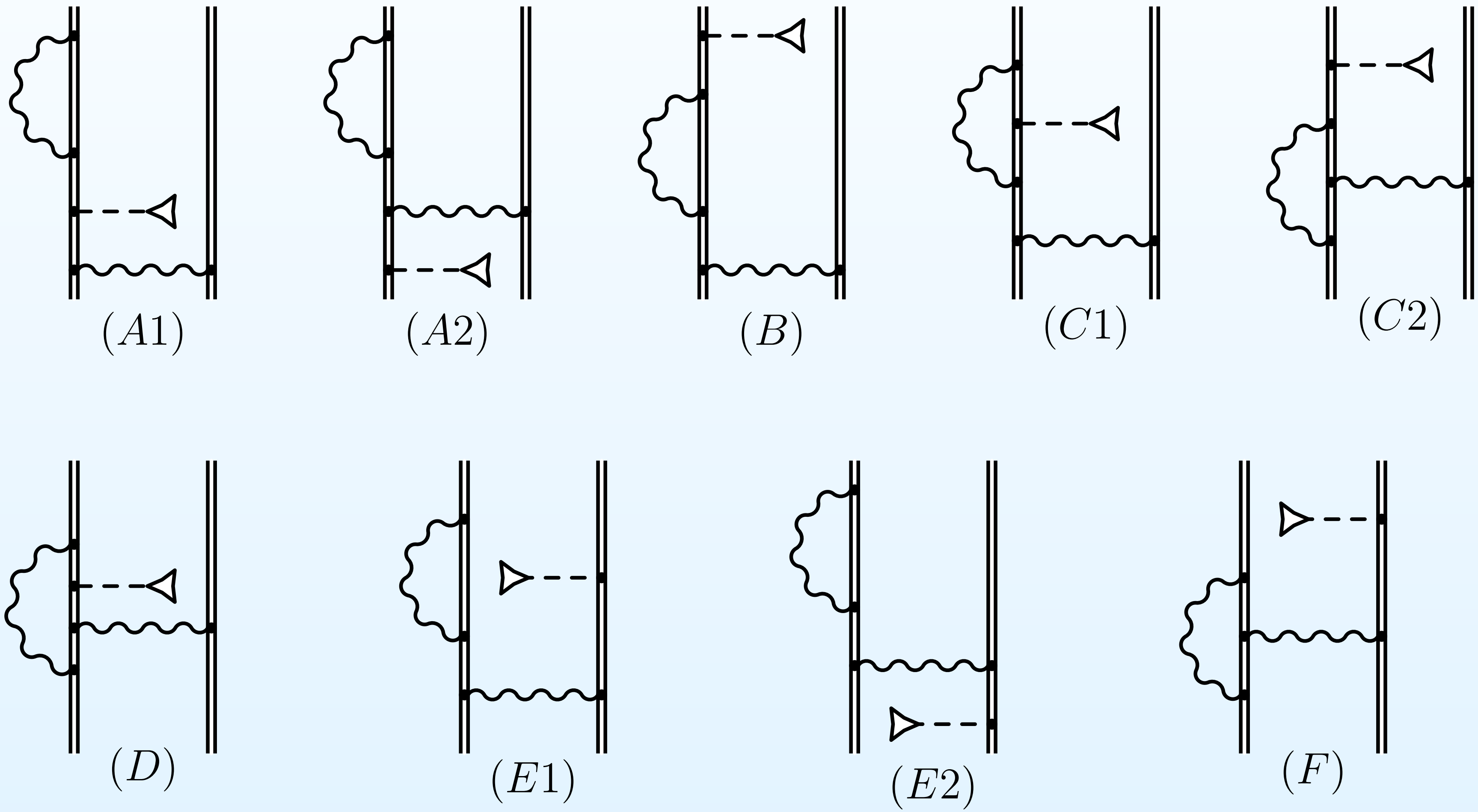
[V. A. Yerokhin et al., PRA (2004)], *[R. N. Lee et al., PRA (2005)]*

Effective screening potential:

[D. A. Glazov et al., PLA (2006)]

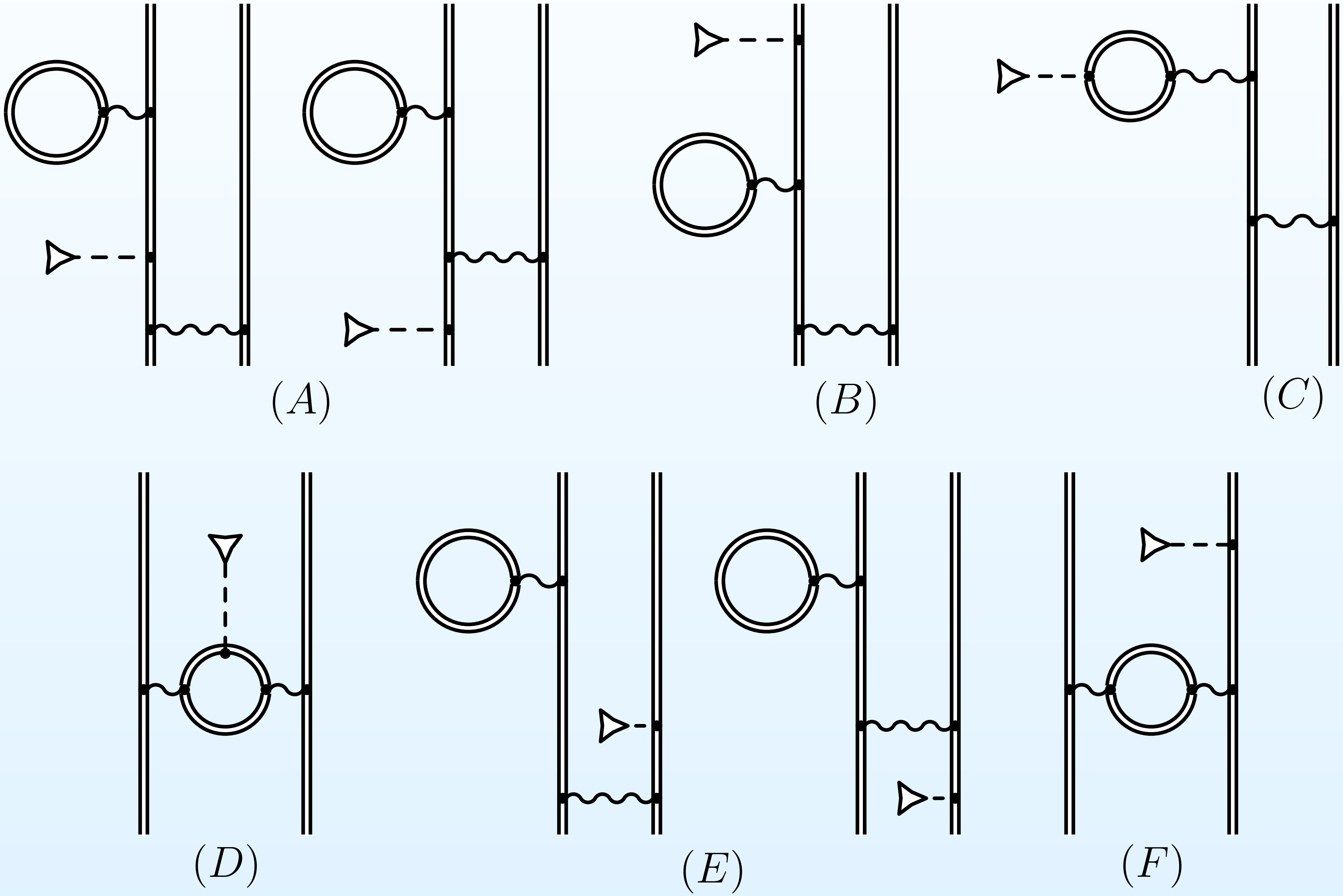
Many-electron one-loop QED

Screened self-energy



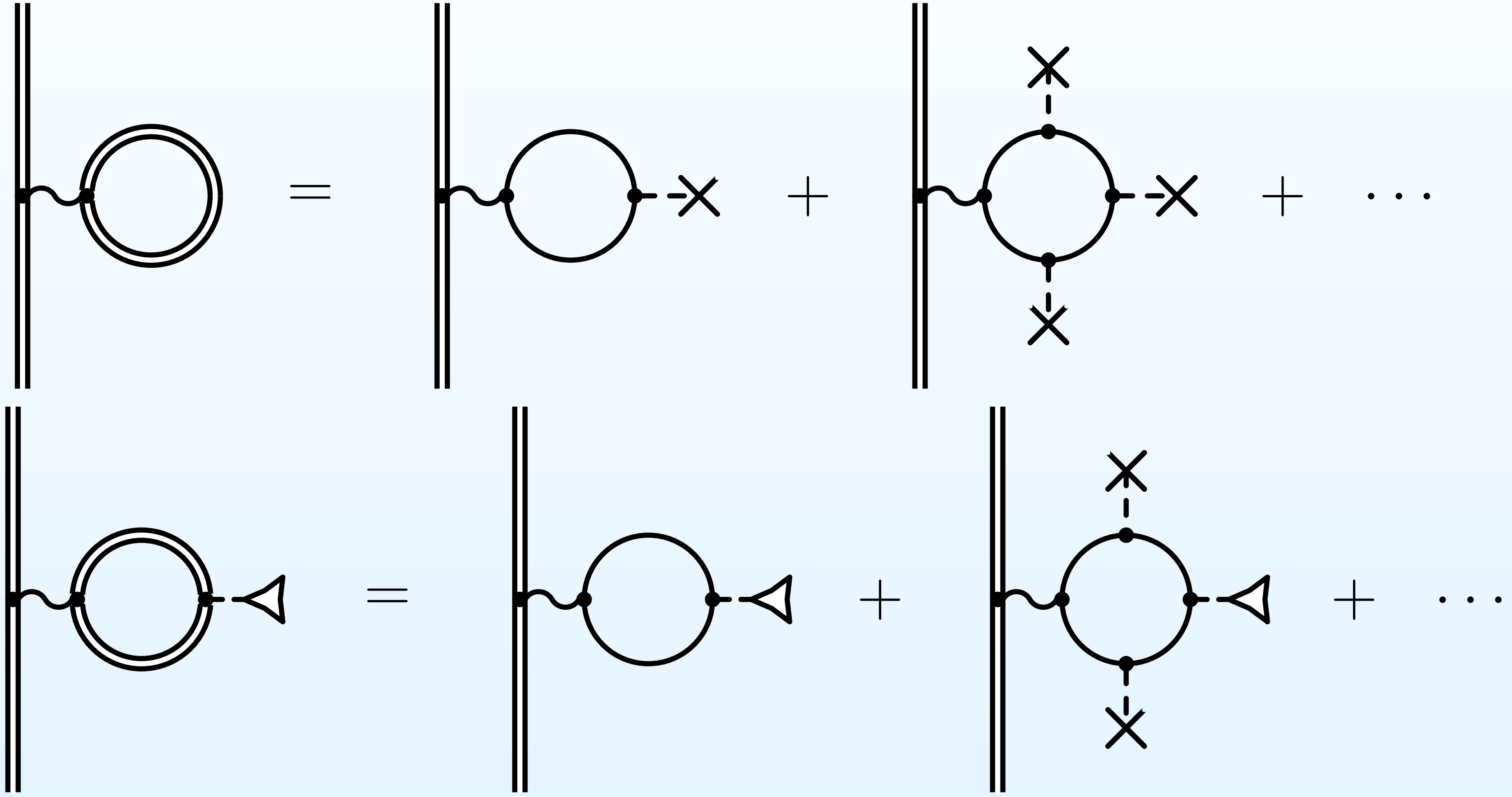
Many-electron one-loop QED

Screened vacuum-polarization



Vacuum polarization

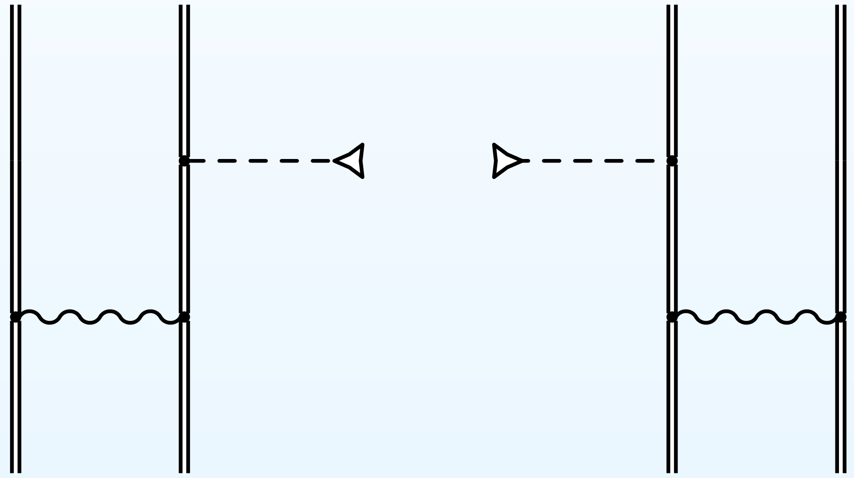
Uehling and Wichmann-Kroll parts



Interelectronic interaction

$$\Delta g_{\rm int} = \frac{1}{Z} B(\alpha Z) + \frac{1}{Z^2} C(\alpha Z) + \ldots$$

$\mathbf{\frac{1}{Z}}$ **:**



$1/Z^2$ and higher:

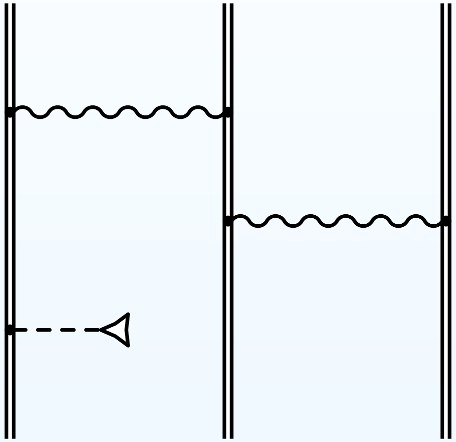
large-scale configuration-interaction Dirac-Fock-Sturm method

Basis: $12s\ 11p\ 10d\ 6f\ 4g\ 2h\ 1i$

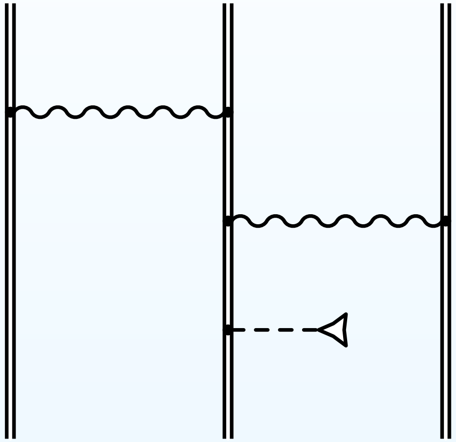
[V. M. Shabaev et al., PRA (2002)], [D. A. Glazov et al., PRA (2004)]

Two-photon exchange

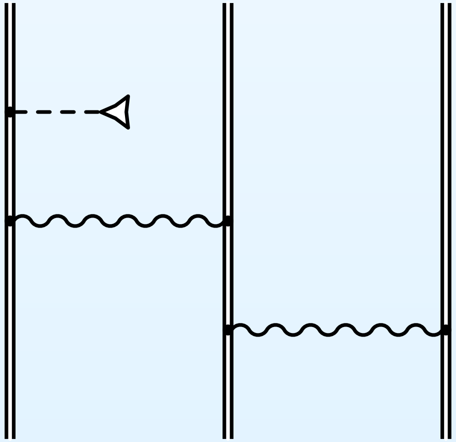
Three-electron diagrams



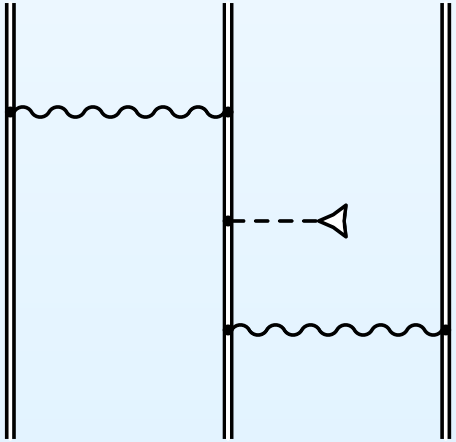
$(1L)$



$(2L)$



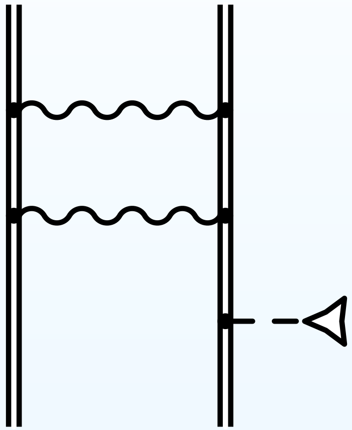
$(1R)$



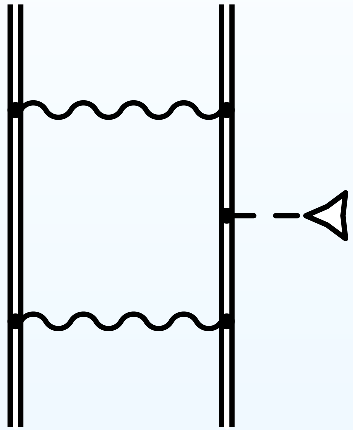
(S)

Two-photon exchange

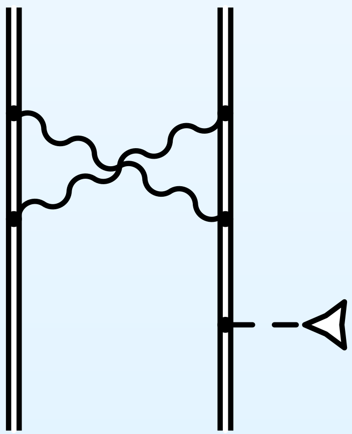
Two-electron diagrams



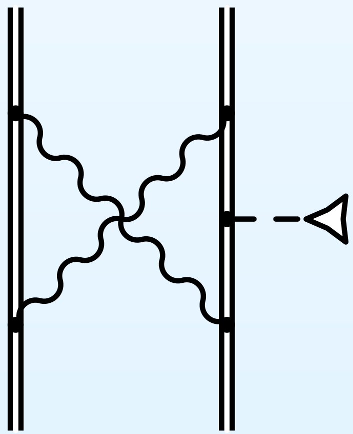
(ladder$-L$)



(ladder$-S$)



(cross$-L$)



(cross$-S$)

g factor of Li-like U

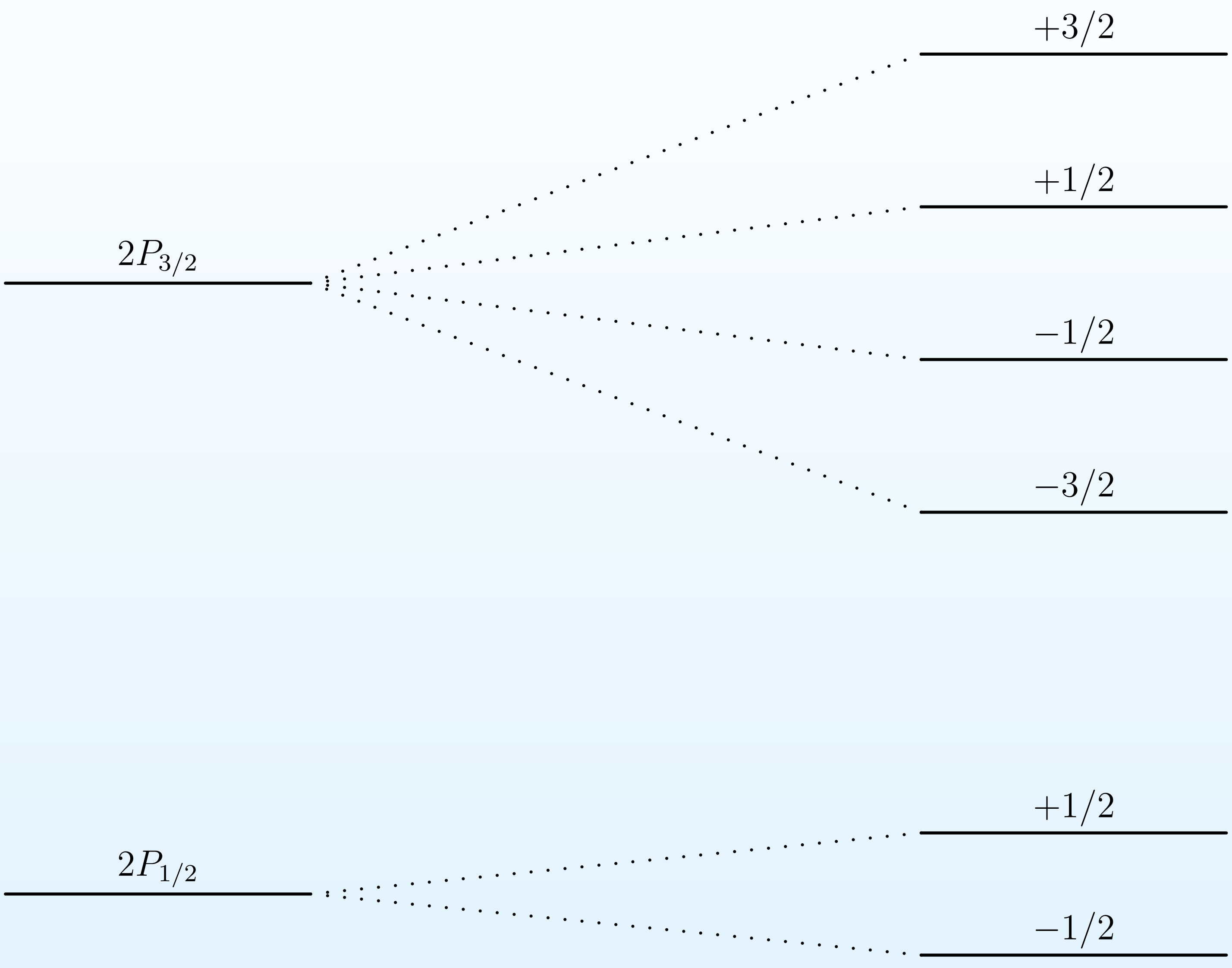
Dirac value (point nucleus)	1.910 722 624
e^{-}–e^{-} interaction, $1/Z$	0.002 509 84
e^{-}–e^{-} interaction, $1/Z^{2+}$	0.000 008 5 (38)
QED, one-loop	0.002 446 3 (2)
QED, two-loop	-0.000 003 6 (8)
QED, screening	-0.000 001 7 (1)
Recoil	0.000 000 3 (7)
Nuclear size	0.000 241 3 (4)
Nuclear polarization	-0.000 000 2 7 (14)
Total theory	1.915 904 9 (40)

[A. V. Volotka et al., PRL (2009)]

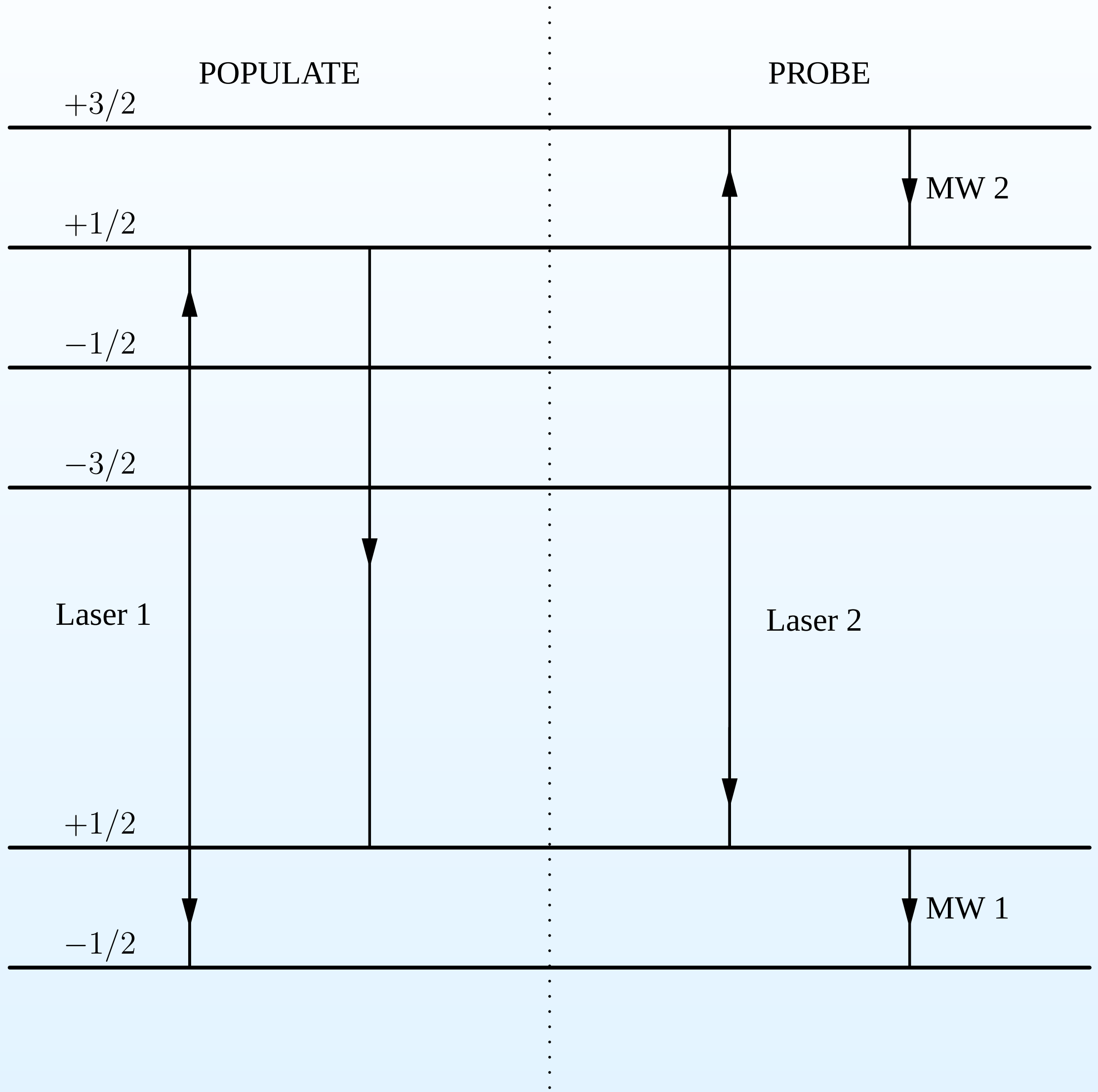
[D. A. Glazov et al., PRA (2010)]

[O. V. Andreev et al., to be published]

B-like ions. Energy levels



B-like ions. Measurement scheme



[M. Vogel and W. Quint, Phys. Rep. (2010), D. von Lindenfels, Diploma thesis (2011)]

g factor of B-like Ar

Dirac value	0.663 775 447
e$^-$–e$^-$ interaction, $1/Z$	0.000 657 525
e$^-$–e$^-$ interaction, $1/Z^{2+}$	$-$0.000 007 5 (8)
QED, one-loop	$-$0.000 768 9 (1)
QED, two-loop	0.000 001 2 (2)
QED, screening	$-$0.000 001 0 (5)
Recoil, NMS	$-$0.000 018 2 (4)
Recoil, SMS	0.000 008 4 (2)
Total theory	0.663 647 (1)

[D. A. Glazov et al., to be published]

Non-linear effects

$$\begin{aligned}
\Delta E &= \Delta E^{(1)} + \Delta E^{(2)} + \Delta E^{(3)} + \ldots \\
\Delta E^{(1)} &= g\, \mu_0 B M_J \\
\Delta E^{(2)} &= g_2 (\mu_0 B M_J)^2 \\
\Delta E^{(3)} &= g_3 (\mu_0 B M_J)^3
\end{aligned}$$

B-like Ar:

$$\begin{aligned}
\Delta E_{\rm Z} = E[+1/2] - E[-1/2] &= 0.269 \cdot 10^{-3}\ \text{eV} \\
\Delta E_{\rm FS} = E[2P_{3/2}] - E[2P_{1/2}] &= 2.814\ \text{eV} \\
\Delta E^{(2)}/\Delta E^{(1)} \sim \Delta E_{\rm Z}/\Delta E_{\rm FS} &\sim 10^{-4} \\
\Delta E^{(3)}/\Delta E^{(1)} \sim (\Delta E_{\rm Z}/\Delta E_{\rm FS})^2 &\sim 10^{-8}
\end{aligned}$$

Collaboration

St. Petersburg

Oleg V. Andreev
Mikhail M. Sokolov
Arseny A. Schepetnov
Vladimir M. Shabaev
Ilja I. Tupitsyn

Dresden

Andrey V. Volotka
Günter Plunien

GSI

David von Lindenfels
Wolfgang Quint
Manuel Vogel
Thomas Stöhlker